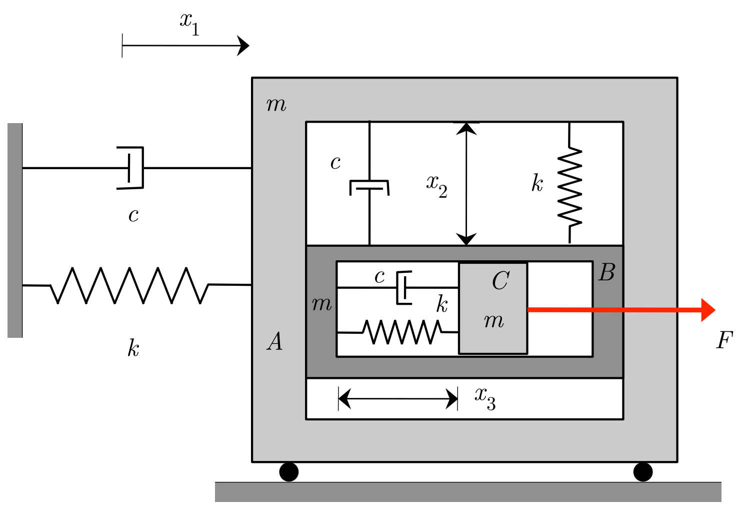
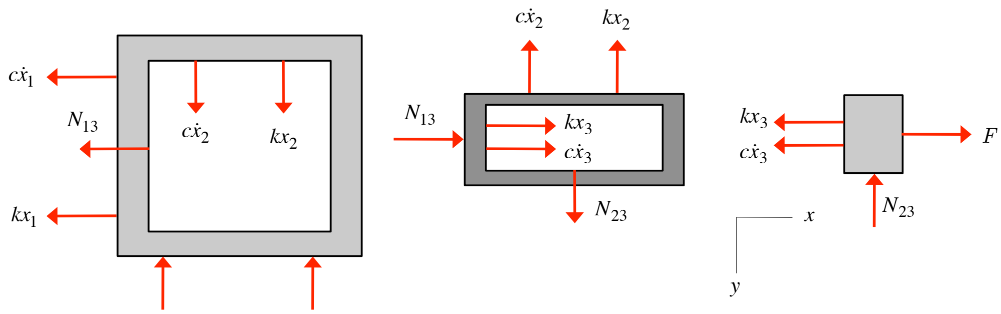
Example A1.4

Given: Consider the three-degree-of-freedom system shown below made up of three particles A, B and C, each of mass m with the system moving within a horizontal plane. Let x_1 describe the absolute motion of particle A, x_2 describe the motion of particle B relative to A and x_3 describe the motion of particle C relative to B. All springs are unstretched when $x_1 = x_2 = x_3 = 0$. Assume all surfaces to be smooth.

Find: For this problem:
a) Draw individual free body diagrams of each particle.
b) Use the Newton-Euler formulation to derive the three differential equations of motion for the system. Your final equations should not include any forces of reaction.
c) Write the equations on motion derived in b) in matrix form. Identify the mass, damping and stiffness matrices in these equations



Free body diagrams:



<u>*Body A*</u>: $\sum F_x = -kx_1 - c\dot{x}_1 - N_{13} = ma_{1x} = m\ddot{x}_1 \quad \Rightarrow \quad N_{13} = -kx_1 - c\dot{x}_1 - m\ddot{x}_1$ (1)

<u>*Body B*</u>: $\sum F_x = kx_3 + c\dot{x}_3 + N_{13} = ma_{2x} = m\ddot{x}_1 \quad \Rightarrow \quad N_{13} = m\ddot{x}_1 - kx_3 - c\dot{x}_3$ (2)

$\sum F_y = -kx_2 - c\dot{x}_2 + N_{23} = ma_{2y} = m\ddot{x}_2 \quad \Rightarrow \quad N_{23} = m\ddot{x}_2 + kx_2 + c\dot{x}_2$ (3)

<u>*Body C*</u>: $\sum F_x = -kx_3 - c\dot{x}_3 + F = ma_{3x} = m\left(\ddot{x}_1 + \ddot{x}_3\right) \quad \Rightarrow \quad \boxed{m\left(\ddot{x}_1 + \ddot{x}_3\right) + c\dot{x}_3 + kx_3 = F}$ (4)

$\sum F_y = -N_{23} = ma_{Cy} = m\ddot{x}_2$ (5)

Equating equations (1) and (2):

$$-kx_1 - c\dot{x}_1 - m\ddot{x}_1 = m\ddot{x}_1 - kx_3 - c\dot{x}_3 \quad \Rightarrow \quad \boxed{2m\ddot{x}_1 + c\dot{x}_1 - c\dot{x}_3 + kx_1 - kx_3 = 0} \qquad (6)$$

Combining equations (3) and (5):

$$m\ddot{x}_2 + kx_2 + c\dot{x}_2 = -m\ddot{x}_2 \quad \Rightarrow \quad \boxed{2m\ddot{x}_2 + kx_2 + c\dot{x}_2 = 0} \qquad (7)$$

Equations (4), (6) and (7) are the three EOMs for the system. Writing these in matrix form gives:

$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & 2m & 0 \\ m & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c & 0 & -c \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} k & 0 & -k \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F \end{Bmatrix}$$

Note that the mass, damping and stiffness matrices are not symmetric.