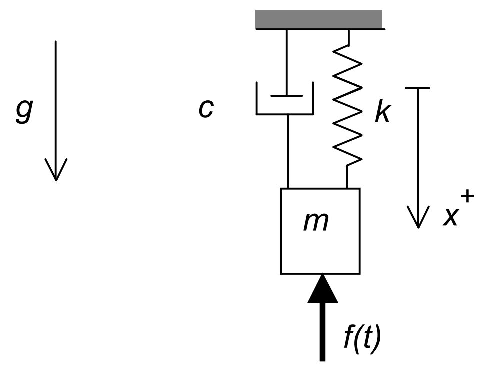
Example A1.1

Given: A particle of mass m is supported by a spring of stiffness k and damping constant c. A vertical force $f(t)$ acts on the particle as shown. Let x describe the position of the particle, where x is measured from the position of the particle when the spring is unstretched.

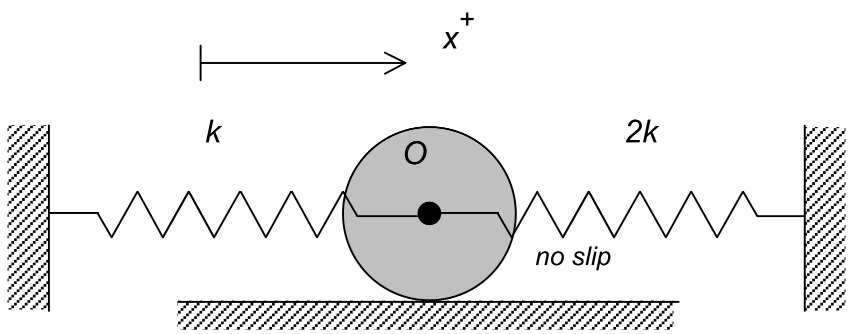
Find: The EOM of this system in terms of the coordinate x.



Example A1.2

Given: A homogeneous disk of mass m and radius r rolls without slipping on a rough horizontal surface. Two springs, having stiffnesses of k and $2k$, are attached between the disk center O and ground, as shown below. Let x describe the position of O, where the springs are unstretched when $x = 0$.

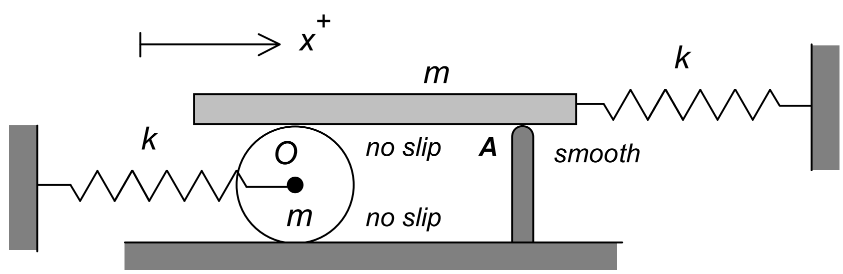
Find: The EOM for the disk in terms of the coordinate x.



Example A1.3

Given: A homogeneous disk of mass of m and radius r rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. A block, also of mass m, is in *no-slip* contact with the top surface of the disk and with a smooth vertical support at A. A second spring of stiffness k is connected between the block and ground. Let x describe the position of O, where the springs are unstretched when $x = 0$.

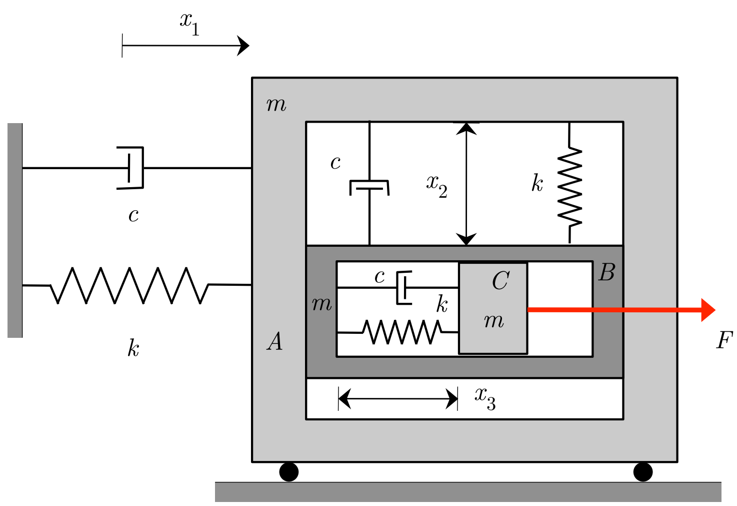
Find: The EOM for the disk in terms of the coordinate x.



Example A1.4

Given: Consider the three-degree-of-freedom system shown below made up of three particles A, B and C, each of mass m with the system moving within a horizontal plane. Let x_1 describe the absolute motion of particle A, x_2 describe the motion of particle B relative to A and x_3 describe the motion of particle C relative to B. All springs are unstretched when $x_1 = x_2 = x_3 = 0$. Assume all surfaces to be smooth.

Find: For this problem:
a) Draw individual free body diagrams of each particle.
b) Use the Newton-Euler formulation to derive the three differential equations of motion for the system. Your final equations should not include any forces of reaction.
c) Write the equations on motion derived in b) in matrix form. Identify the mass, damping and stiffness matrices in these equations



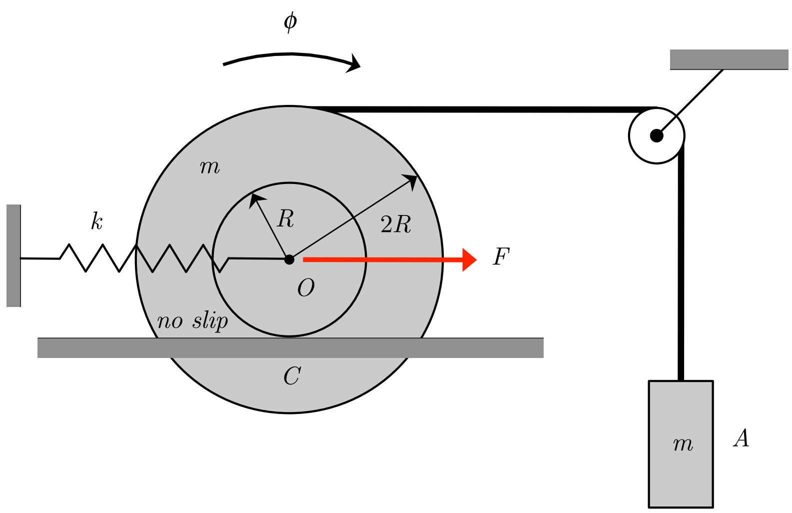
Example A1.5

Given: The stepped-spool below has a mass of m and amass moment of inertia of I_O about point O. Let ϕ represent the angle of rotation of the disk with the spring being unstretched when $\phi = 0$.

Find: For this problem:

a) Using the Newton-Euler formulation, determine the equation of motion for the system in terms of the coordinate ϕ. Draw the free body diagrams of the drum and block individually before writing down the Newton-Euler equations.

b) Write the equations on motion derived in b) in matrix form. Identify the mass, damping and stiffness matrices in these equations



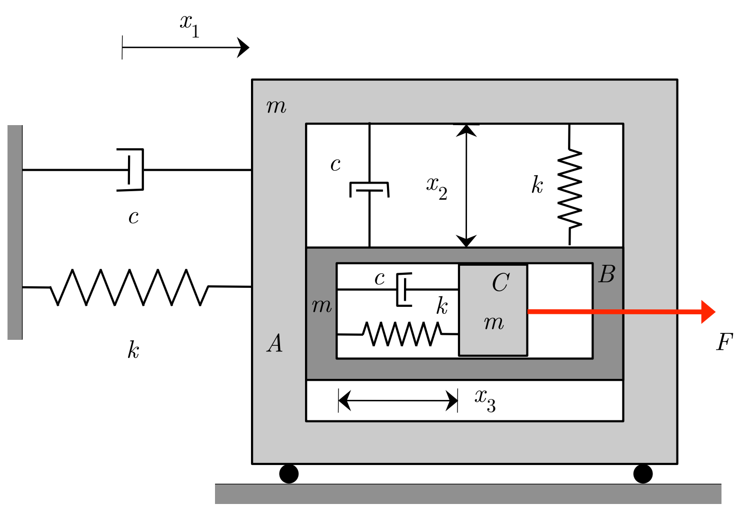
Example A1.6

Given: Consider the three-degree-of-freedom system shown below made up of three particles A, B and C, each of mass m with the system moving within a horizontal plane. Let x_1 describe the absolute motion of particle A, x_2 describe the motion of particle B relative to A and x_3 describe the motion of particle C relative to B. All springs are unstretched when $x_1 = x_2 = x_3 = 0$. Assume all surfaces to be smooth.

Find: For this problem:

a) Determine the generalized mass coefficients, m_{ij}, for the system corresponding to the generalized coordinates x_1, x_2 and x_3.

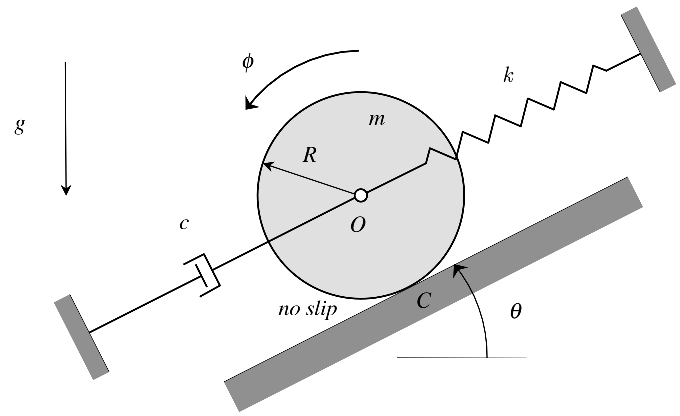
b) Determine the generalized force, Q_i, for the system corresponding to the generalized coordinates x_1, x_2 and x_3.



Example A1.7

Given: A homogeneous disk of mass m and outer radius R is able to roll without slipping on an inclined ramp. A spring (of stiffness k) and a dashpot (of damping constant c) connect the center of the disk O to ground. The spring is unstretched when $\phi = 0$.

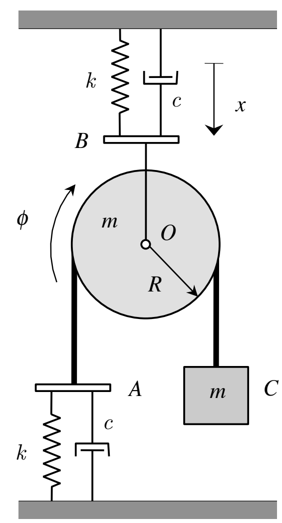
Find: Use Lagrange's equations to derive the EOM for this single-DOF system in terms of the generalized coordinate ϕ.



Example A1.8

Given: A homogeneous disk of mass m and outer radius R is supported by an in-parallel connection of a spring (of stiffness k) and of a dashpot (of damping coefficient c). An inextensible cable is wrapped around the outer perimeter of the disk. One end of the cable is attached to a second, in-parallel spring/dashpot connection, with the other end attached to block C (of mass m). Let x represent the motion of the massless connector B, and ϕ the rotation of the disk. Let $\phi = 0$ when the springs are unstretched. Assume that the cable does not slip on the disk. All motion of the system occurs in a horizontal plane.

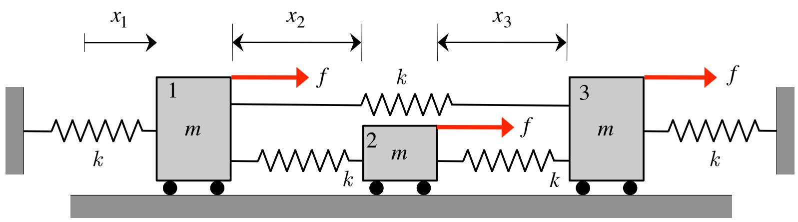
Find: Use Lagrange's equations to derive the EOM for this single-DOF system in terms of the generalized coordinate ϕ.



Example A1.9

Given: Three blocks, each of mass m, are able to move along a smooth horizontal surface. The blocks are interconnected by springs, as shown in the figure. Identical forces f act to the right on each of the blocks. Let $x_1$represent the absolute motion of block 1, x_2 represent the motion of 2 relative to 1 and x_3 represent the motion of 3 relative to 2.

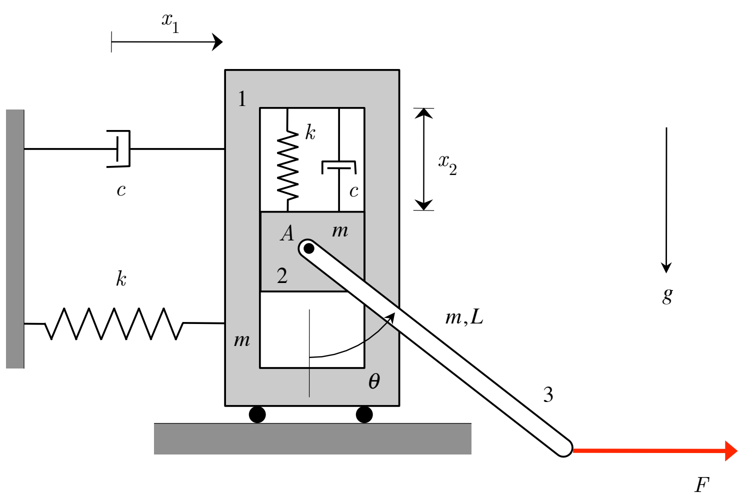
Find: Use Lagrange?s equations to derive the EOMs for this three-DOF system in terms of the generalized coordinates x_1, x_2 and x_3.



Example A1.10

Given: The system shown is made of bodies 1, 2 and 3, with each body having a mass of m. Body 1 is constrained to move along a smooth horizontal floor. Body 2 is constrained to move within a vertical slot in body 1. Body 3 (a thin, homogeneous bar) is pinned to body 2 at its end A. The coordinates x_1, x_2 and θ are used to describe the position and orientation of the bodies in the system. x_1 is an absolute coordinate, x_2 describes the motion of 2 relative to 1, and θ measures the rotation of body 3 from its downward orientation.

Find: Use Lagrange?s equations to derive the EOMs for this system in terms of the coordinates x_1, x_2 and θ.



Example A1.11

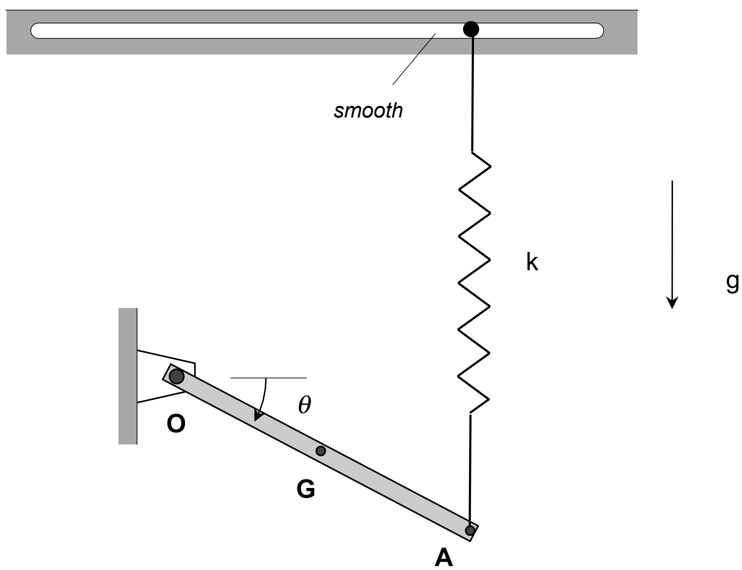
Given: The thin homogeneous bar shown has a length of L and mass of m. A spring is attached to end A of the bar, with the other end of the spring able to slide in a smooth horizontal slot. Note that having the spring slide in the slot permits the spring to remain vertical for all motion. The spring is unstretched when $\theta = 0$.

Find: For this problem:

a) Determine the equilibrium angle θ_0 for the system. You will find multiple equilibrium angles, including $\pm 90°$ corresponding to the vertical positions of the bar. Consider only the non-vertical equilibrium angle.

b) Determine the linearized EOM of the system for small motion $z = \theta - \theta_0$.

c) What is the natural frequency for small oscillations about the equilibrium state corresponding to $mg/kL = 1.2$?



Example A1.12

Given: The absolute coordinates y_1, y_2 and y_3 are used to describe the motion of A, B and the center of mass G of the homogeneous wheel.

Find: For this problem:

a) Write down the potential energy function U for the system. Use the following equation to find the stiffness matrix for the system:

$$K_{ij} = \left[\frac{\partial^2 U}{\partial q_i \partial q_j} \right]_{q_0}$$

b) Find the flexibility matrix [A] using the influence coefficient method.

c) Using $[K]$ and $[A]$ from above, verify that $[A][K] = [I]$.