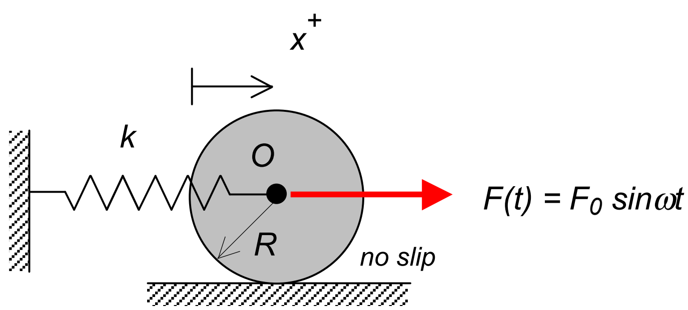
Example A4.1

Given: A homogeneous disk having a mass of m and radius R rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. Let x describe the position of O, where the spring is unstretched when $x = 0$.

Find:

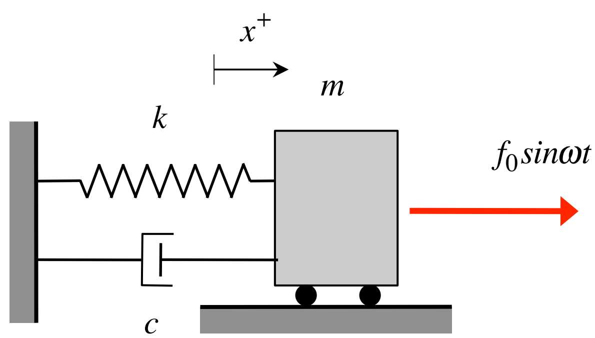
1. An EOM for the disk in terms of the coordinate x.
2. The particular solution of the EOM.
3. The amplitude of response corresponding to $\omega = 15$ rad/s. Is the response in phase or out of phase with the forcing? Use $m = 2$ kg, $k = 300$ N/m and $F_0 = 20$ N.



Example A4.2

Given: A pressure of $p = 0.625 \cos 30t$ psi acts on piston with cross-sectional area of $A = 80$ in^2.

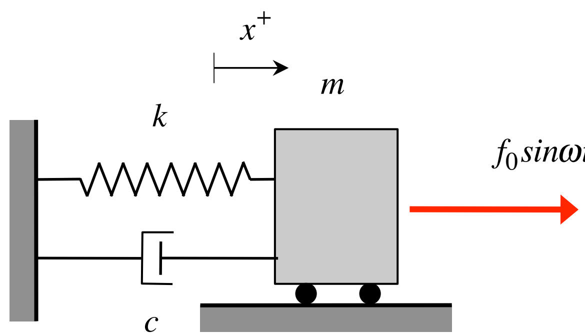
Find: The steady-state response of the piston and the maximum force transmitted to the base. Use $mg = 100$ lb, $k = 2400$ lb/ft and $c = 85$ lb-s/ft.



Example A4.3

Given: The frequency of excitation is changed from ω_1 to ω_2. As a result, the amplitude of steady-state response goes from X_1 to X_2, with $X_2 = X_1/8$.

Find: The damping ratio ζ of the system.

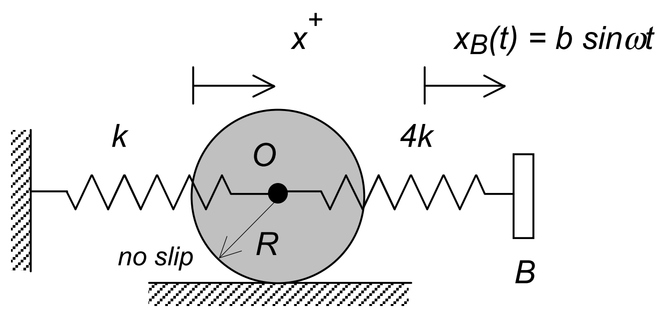


Example A4.4

Given: The system shown below.

Find:

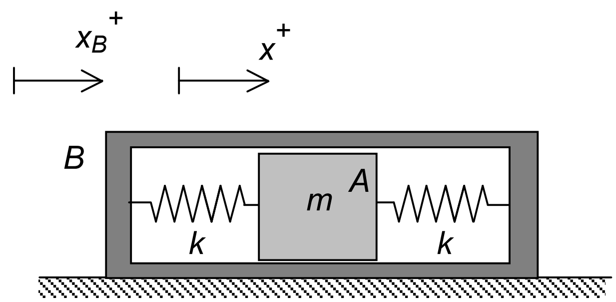
1. The EOM for the disk in terms of the coordinate x.
2. The particular solution of the EOM.
3. The amplitude of response corresponding to $\omega = 15$ rad/s. Is the response in phase or out of phase with the forcing? Use $m = 4$ kg, $k = 800$ N/m and $b = 20$ mm.



Example A4.5

Given: Box B is given a prescribed displacement of $x_B(t) = b \sin \omega t$.

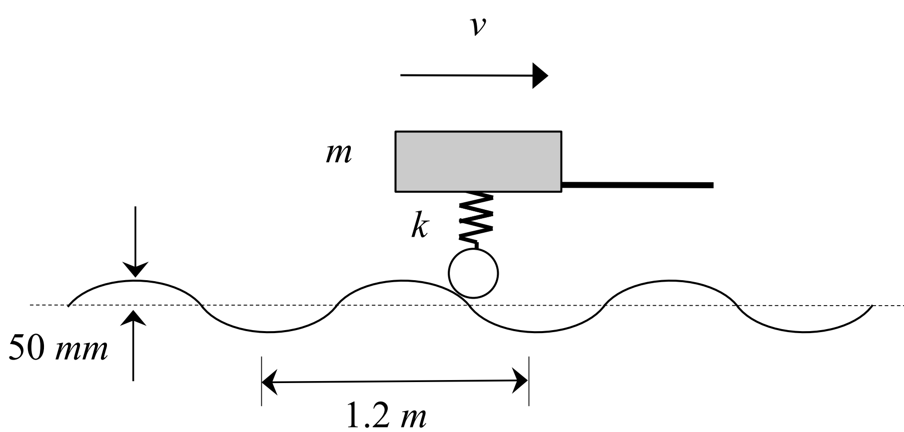
Find: The two values of frequency ω for which the amplitude of forced response of the block A is twice that of the box B.



Example A4.6

Given: Adding 75 kg of mass to the trailer creates 3 mm of additional sag in the suspension.

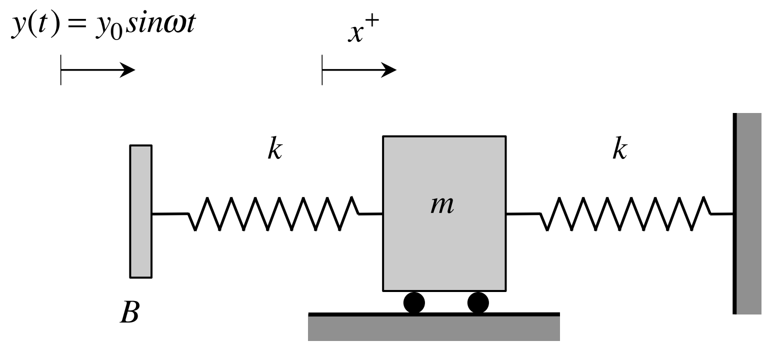
Find: The vibration amplitude X and critical speed at which the undamped oscillations are the greatest as the trailer is pulled along a roadway with sinusoidal waviness. Use $m = 400$ kg and $v = 25$ km/hr.



Example A4.7

Given: The instrument shown below has a mass of 50 kg and is spring-mounted to a base. This base experiences a prescribed harmonic motion of amplitude of 0.20 mm and frequency ω.

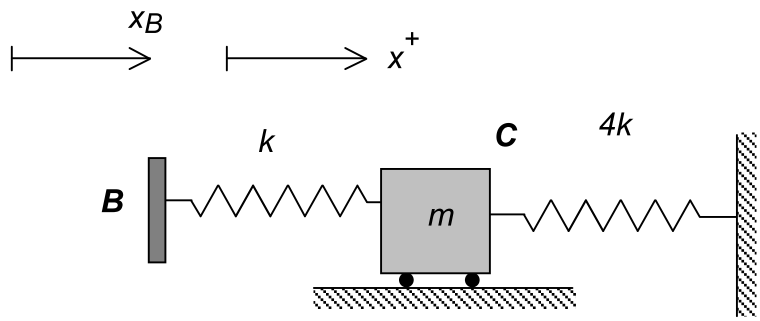
Find: The range of frequencies ω for which the amplitude of the oscillatory motion of the instrument does not exceed 0.30 mm. The total stiffness of the mount is 30 kN/m.



Example A4.8

Given: Determine the differential equation of motion for the system shown where $x_B(t) = b \sin \omega t$.

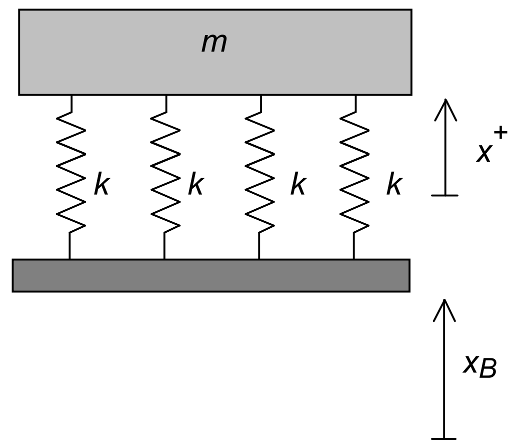
Find: The particular solution of this equation of motion.



Example A4.9

Given: The system shown below.

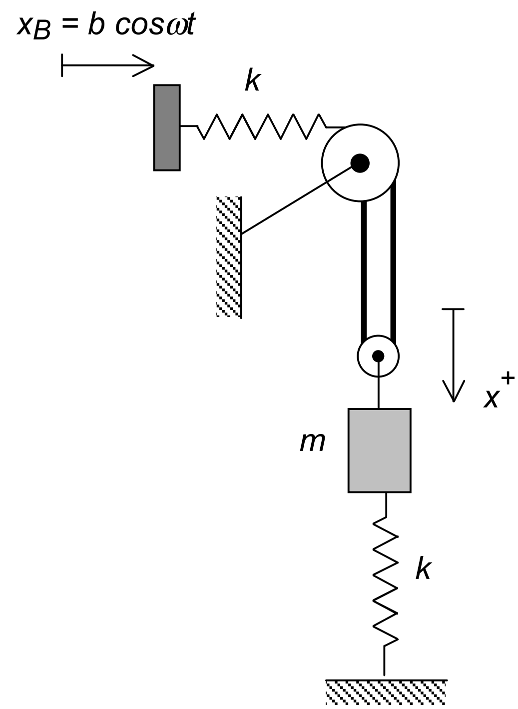
Find: The differential equation of motion for the system shown and the amplitude of the steady-state response of the system. Use $m = 50$ kg, $k = 7500$ N/m and $x_B(t) = 0.002 \sin 50t$.



Example A4.10

Given: The system shown below.

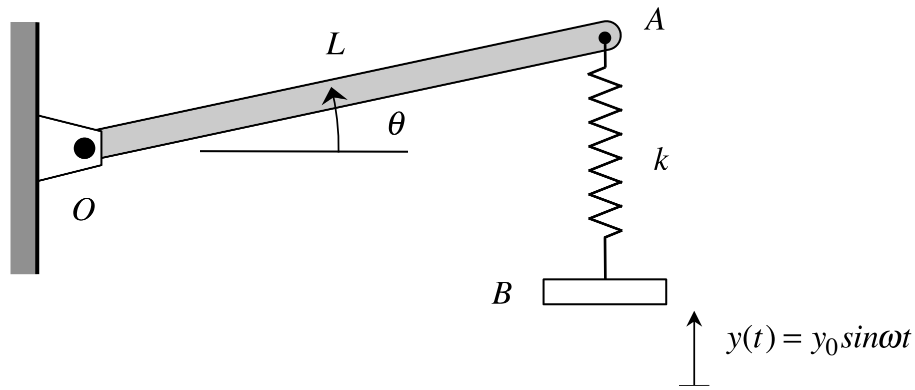
Find: The differential equation of motion for the system shown and the resonant frequency.



Example A4.11

Given: Two identical, thin bars (each of mass m and length L) are welded together at a right angle. This T-shaped assembly is pinned to ground at O. Block B is given a prescribed vertical displacement of $x_B(t) = b \sin \omega t$.

Find: The differential equation of motion for the system in terms of the rotation angle θ. Assume small oscillations.

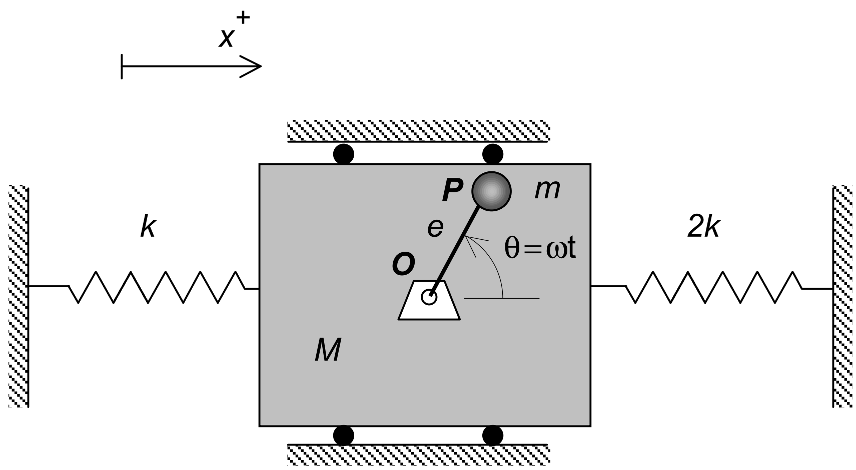


HORIZONTAL PLANE

Example A4.12

Given: Consider the system shown below with a rotating imbalance excitation.

Find: The equation of motion for this system in terms of the coordinate x. Determine the amplitude of the steady-state response $x(t)$ for the system. Use the following parameters: $m = 0.1$ kg, $M = 4.9$ kg, $k = 1500$ N/m, $e = 20$ mm and $\omega = 40$ rad/s.



Example A4.13

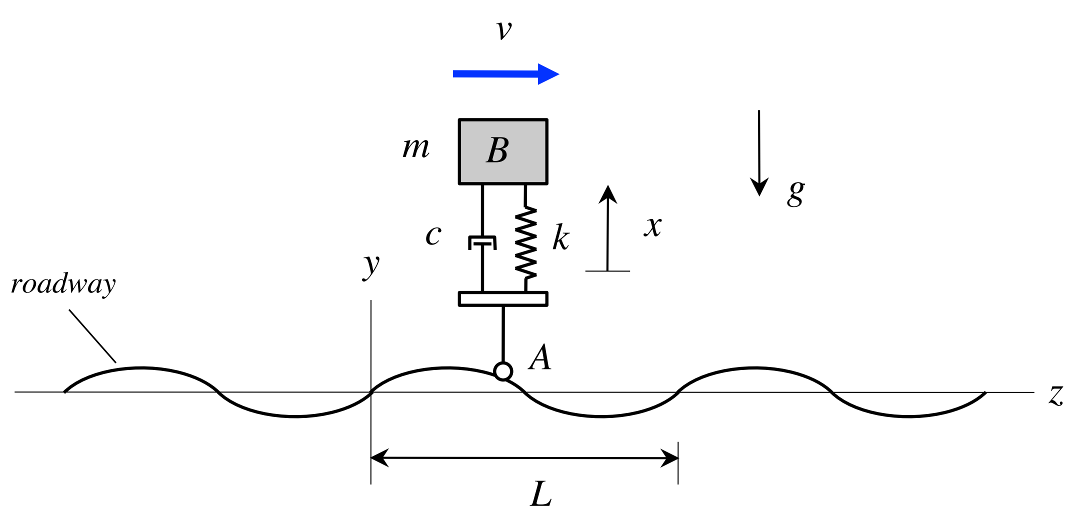
Given: Consider the single-DOF model of an automotive system shown below as the automobile moves along a wavy-surfaced roadway with a constant speed of v. The profile of the roadway is given by $y(z) = y_0 sin(\pi z/L)$. Let x represent the vertical motion of the automobile body B as measured from the stretched position of the body. Assume that the wheel A does not lose contact with the roadway surface as the system moves along the road.

Find: For this problem:

a) Derive the equation of motion (EOM) of the system in terms of the coordinate x. Transform this EOM to be in terms of the coordinate $r = x - mg/k$.

b) Write down the particular solution $r_P(t)$for your EOM. Make a hand sketch of the UNDAMPED response amplitude of $r_P(t)$ vs. the temporal excitation frequency. Scale the response amplitude by y_0 and the frequency by $\sqrt{k/m}$.

c) Let $F(t)$ represent the time-varying portion (i.e., excluding the influence of weight) of the force acting on A by the roadway. Derive an expression the particular solution $F_P(t)$. Make a hand sketch of the UNDAMPED response amplitude of $F_P(t)$ vs. the temporal excitation frequency. Scale the response amplitude by ky_0 and the frequency by $\sqrt{k/m}$.

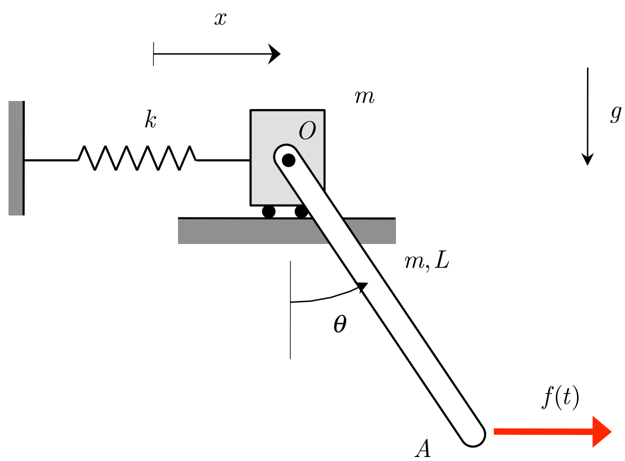


Example A4.14

Given: A forcing $f(t) = f_0 sin\Omega t$ acts at the end of the thin, homogeneous bar of the two-DOF system shown below. The response of the system is to be described by the coordinates $x(t)$ and $\theta(t)$. Let $g/L = 2k/m$.

Find: For this problem:

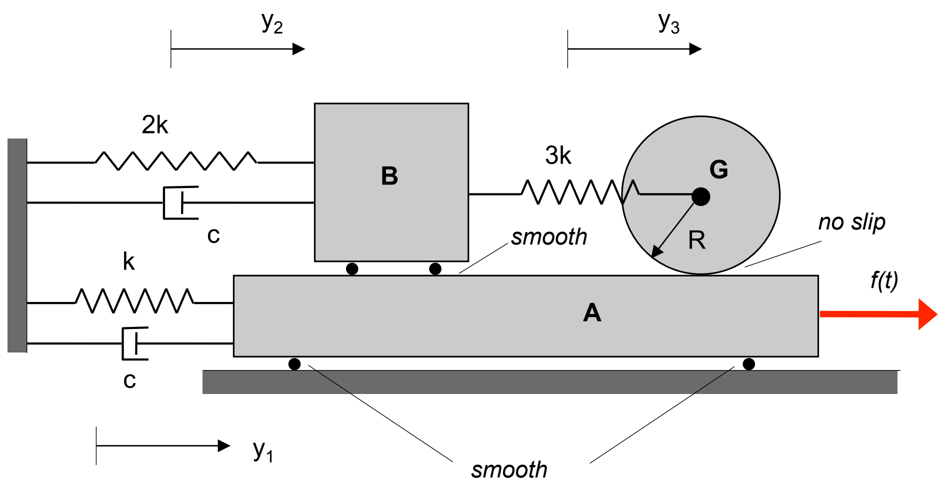
a) Derive the particular solutions $x_P(t)$ and $\theta_P(t)$ for the system.
b) At what values of the temporal frequency Ω does resonance occur in the system?
c) Show that the "shape" of the response is that of the first mode when excited at the first natural frequency, and that the shape is that of the second mode when excited at the second natural frequency.
d) At what values (if any) of the temporal frequency Ω do anti-resonances occur for $x_P(t)$? For $\theta_P(t)$
e) Make hand sketches for the amplitudes of $x_P(t)$ and $\theta_P(t)$ vs. the frequency Ω.



Example A4.15

Given: An external forcing is applied to the system below, where $f(t) = f_0 sin \Omega t$. The response of the system is to be described by the coordinates $y_1(t)$, $y_2(t)$ and $y_3(t)$. The particular solutions for these coordinates are to labeled as $y_{P1}(t)$, $y_{P2}(t)$ and $y_{P3}(t)$.

Find: For this problem:
a) At what values of the frequency Ω does resonance occur in the UNDAMPED system?
b) At what values (if any) of the frequency Ω do anti-resonances occur in the UNDAMPED system for $y_{P1}(t)$? For $y_{P2}(t)$? For $y_{P3}(t)$
c) Using the complex exponential approach, derive the form of the particular solutions $y_{P1}(t)$, $y_{P2}(t)$ and $y_{P3}(t)$ for the DAMPED system. Do not invert the matrix needed for solution.
d) Using Matlab, produce plots for the amplitudes of $y_{P1}(t)$, $y_{P2}(t)$ and $y_{P3}(t)$ vs. the excitation frequency Ω for four values of damping: $c/\sqrt{km} = 0, 0.2, 0.3, 0.4$.



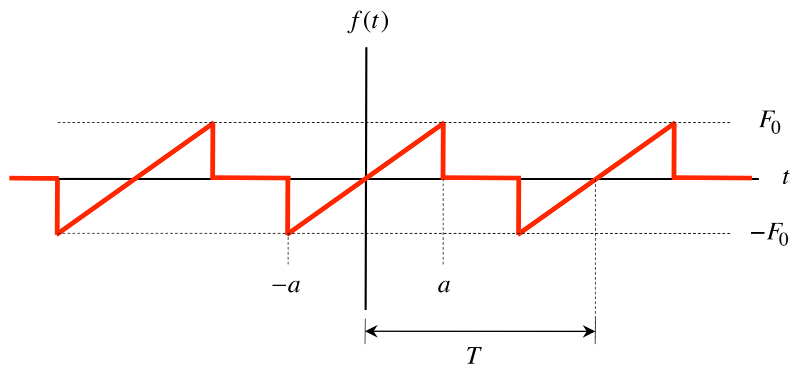
Example A4.16

Given: Consider the T-periodic function f(t) shown below.

Find: For this problem:

a) Determine the Fourier series of $f(t)$. Take advantage of any symmetry/anti-symmetry/zero mean value characteristics of $f(t)$ when finding the Fourier series coefficients.

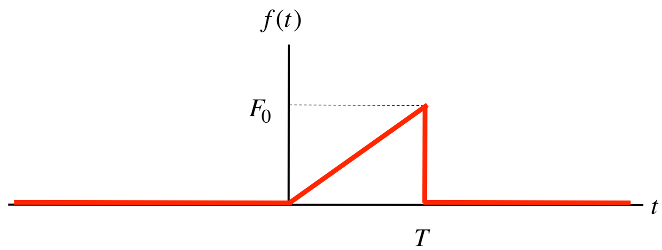
b) Use Matlab, or equivalent application, to plot the truncated Fourier series found in a) above. Use a sufficiently large number of terms in the series when plotting to observe the convergence of this series.



Example A4.17

Given: An undamped oscillator, $m\ddot{x} + kx = f(t)$, has the forcing $f(t)$ shown below. The system has the following initial conditions: $x(0) = \dot{x}(0) = 0$.

Find: For this problem:
a) Using the convolution integral solution method, determine the response $x(t)$ of the system. Clearly indicate the $t < T$ and $t > TT$ components of your solution.
b) If $T = \pi\sqrt{m/k}$, at what time does the first maximum in the response x(t) occur? Provide a graphical explanation/interpretation for your answer.



Example A4.18

Given: Consider the following damped single-DOF oscillator: $m\ddot{x} + c\dot{x} + kx = f(t)$, where $f(t)$ is an arbitrary forcing. It is also known that the system is critically damped.

Find: For this problem, derive the form of the convolution integral solution for this system. Feel free to start with equations (IV.12) and (IV.13) of the lecture book in your derivation.

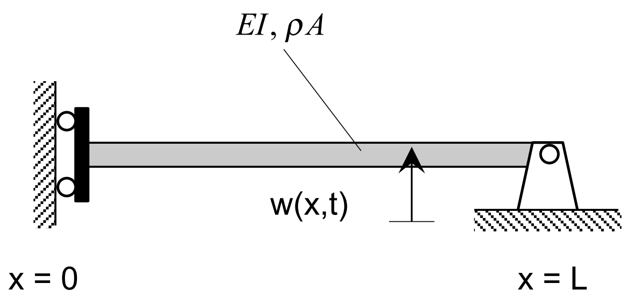
Example A4.19

Given: Consider the bending beam shown below with a concentrated harmonic force acting at the mid-length point C on the beam.

Find: For this problem:

a) Use the modal uncoupling approach to determine the particular solution $w_P(t) = W(x) sin\Omega t$ of the beam's EOM.

b) Make a hand sketch of $|W(x = L/2)|$ vs. Ω for a range of values of Ω covering up through the first four resonances of the response. Provide details on how you arrived at this hand sketch.



Example A4.20

Given: A point load acts on end particle "1" of the beam: $f(t) = I_F \delta(t)$, where I_F is the impulse of the force $f(t)$. The mass of the beam is negligible as compared to the mass of the three particles.

Find: For this problem:

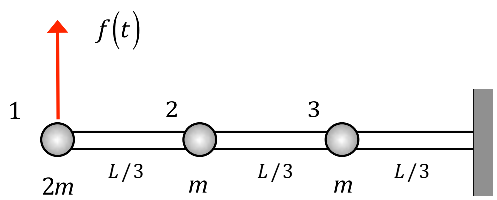
a) Write down the modal EOMs for the system in symbolic form.

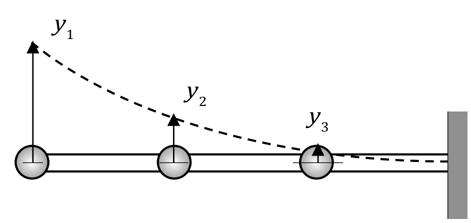
b) Solve the modal equations corresponding to zero initial conditions, again leaving your answer in symbolic form.

c) Write a Matlab (or equivalent) code to find the natural frequencies (scaled by $\sqrt{EI/mL^3}$) and the mass-normalized modal vectors.

d) Based the results in b) and c), comment on the relative size contributions of the three modes to the response.

e) Add to your Matlab code the ability to produce plots of three particle displacements scaled by $I_F/\sqrt{mEI/L^3}$ vs. non-dimensional time, $t\sqrt{EI/ml^3}$.





Example A4.21

Here we reconsider the building problem developed earlier in the course. Here we will mount an eccentric shaker on the top floor of the building to simulate a modal test of the building. Find the response of the building as a function of the shaker frequency Ω. How would the response change if the shaker were instead mounted on the second floor from the top?