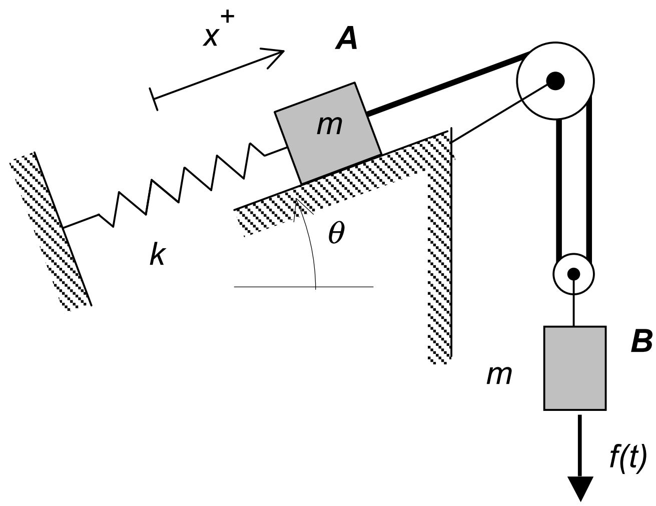
Example A2.1

Given: The system shown below.

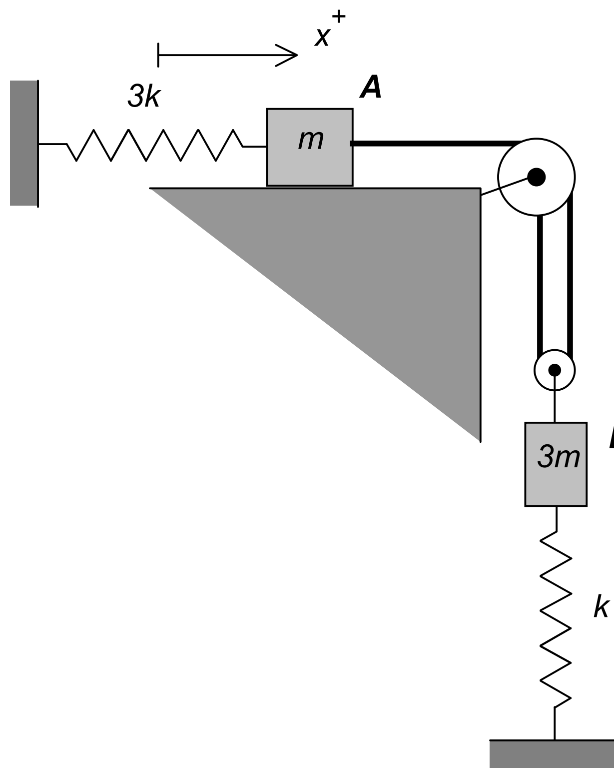
Find: The EOM of this system in terms of the coordinate x and the natural frequency of the system.



Example A2.2

Given: Blocks A and B (having masses of m and $3m$, respectively) are connected by a cable-pulley system as shown below. A spring of stiffness $3k$ is attached between A and ground. A second spring of stiffness k is attached between block B and ground. Let x describe the position of A, where the springs are unstretched when $x = 0$.

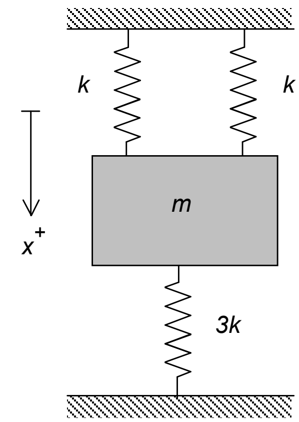
Find: The EOM for the system in terms of the coordinate x and the natural frequency of free oscillations for this system.



Example A2.3

Given: The block has a downward speed of 2 m/s as it passes through its equilibrium position.

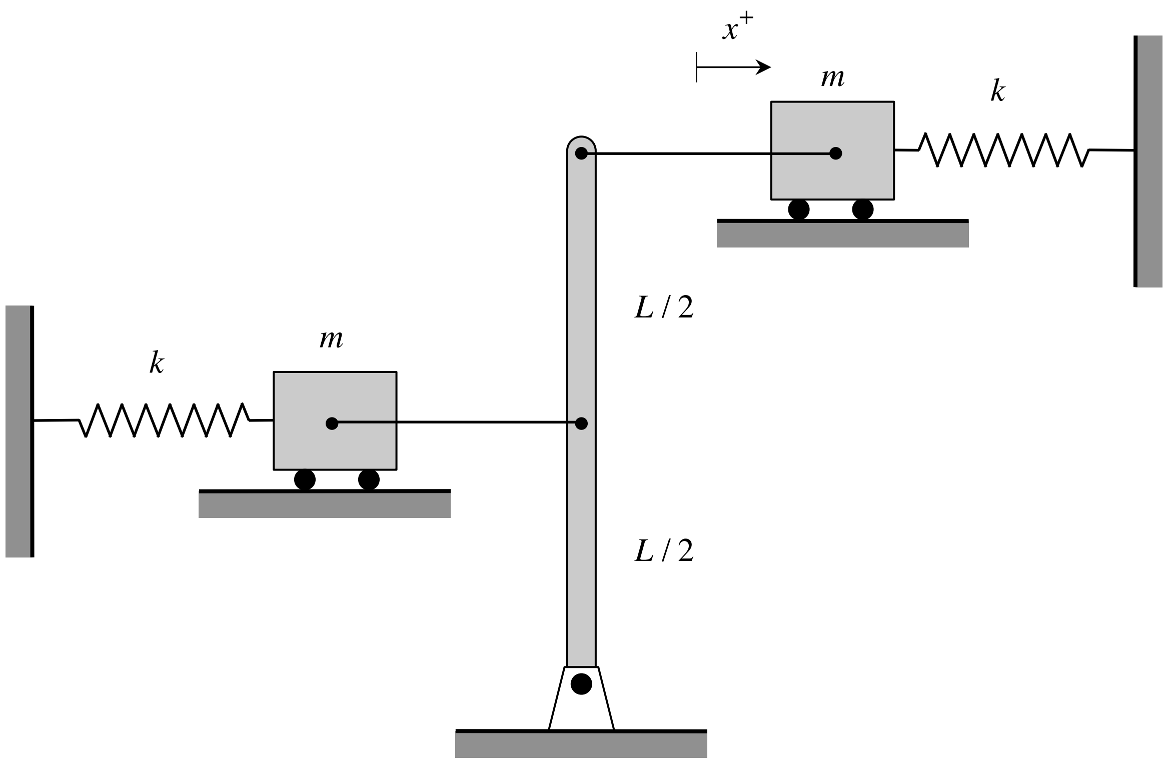
Find: The maximum acceleration of the block over one cycle of oscillation. Use $k = 1000$ N/m and $m = 50$ kg.



Example A2.4

Given: The system shown below.

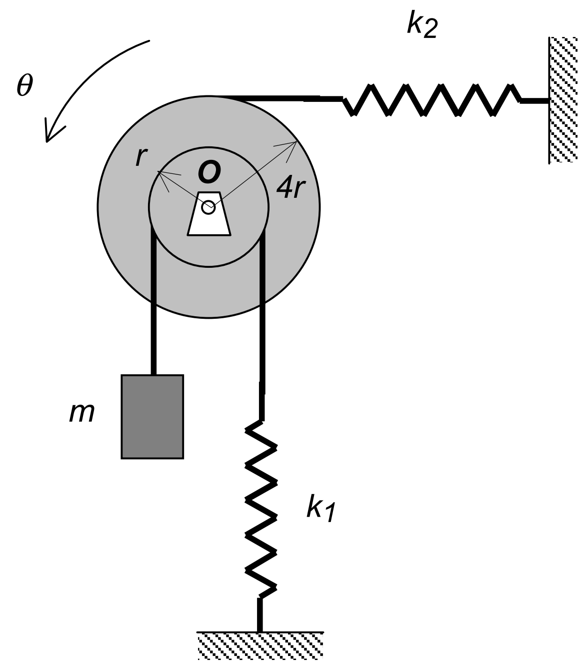
Find: The differential equation of motion for the system using the coordinate x and the natural frequency of the system. Assume small oscillations of the system.



Example A2.5

Given: The pulley has a mass moment of inertia of I_O.

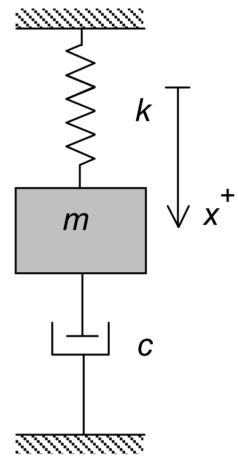
Find: The differential equation of motion for the system in terms of θ and the natural frequency for the system.



Example A2.6

Given: The system shown below.

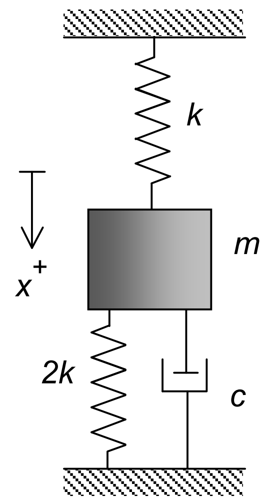
Find: The value of c corresponding to critical damping. Use $k = 30$ kN/m and $m = 35$ kg.



Example A2.7

Given: The system shown below.

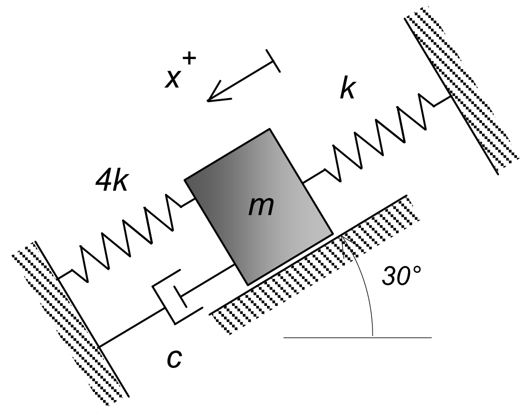
Find: The value of the damping constant c, such that the system has 50 percent of critical damping. Use $k = 3000$ N/m and $m = 10$ kg.



Example A2.8

Given: The system shown below.

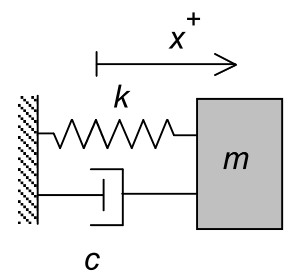
Find: The value of the damping constant c, such that the system has 50 percent of critical damping. Use $k = 2000$ N/m and $m = 10$ kg.



Example A2.9

Given: The system shown below.

Find: The free response of the system corresponding to $x(0) = x_0$ and $\dot{x}(0) = 0$. Use $c = 2.5$ lb-s/ft, $k = 36$ lb/ft and $mg = 8$ lb.



Example A2.10

Given: The addition of damping to an undamped system causes the period to increase by 25 percent.

Find: The value of the damping ratio after the addition of damping.

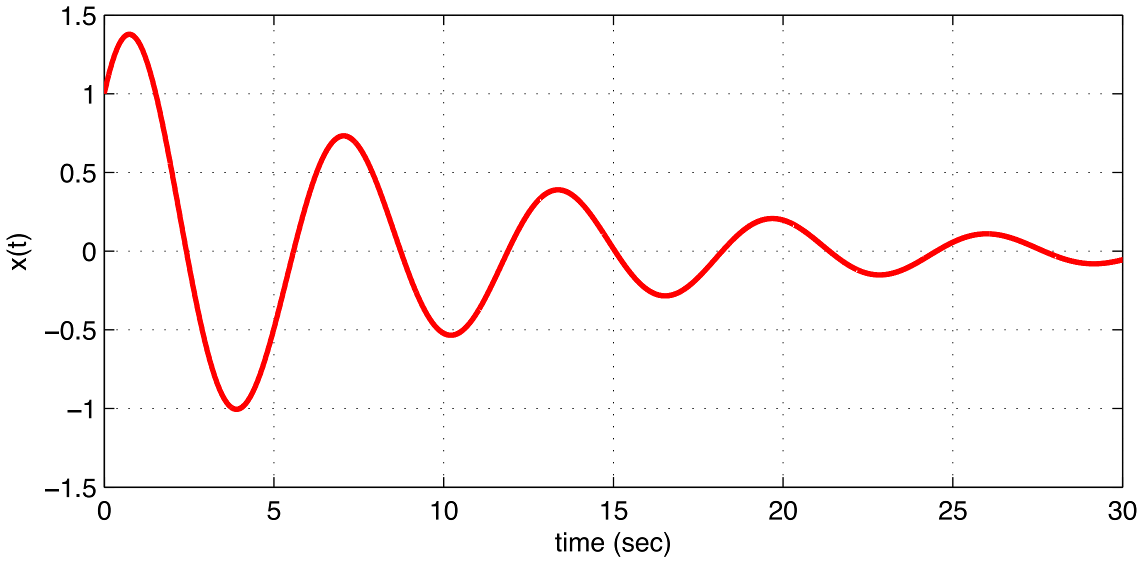
Example A2.11

Given: The free response of the single degree of freedom system:

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

shown below. It is known that $M = 2$ kg.

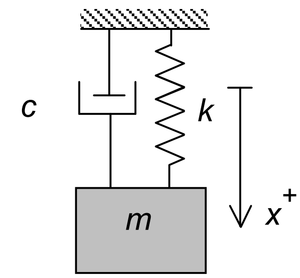
Find: The damping coefficient C from this free response plot.



Example A2.12

Given: The system shown below is released from rest under the action of gravity.

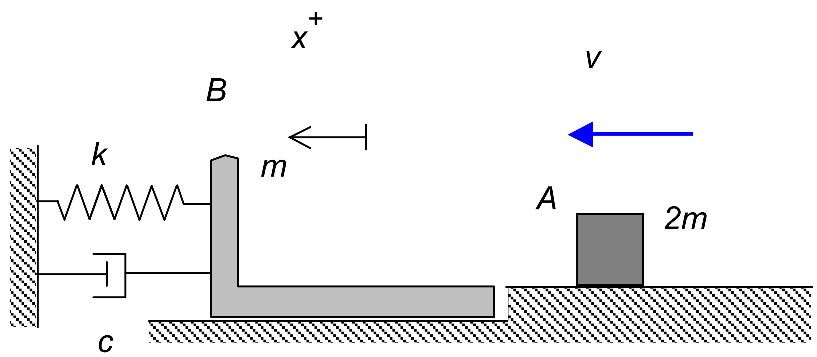
Find: The initial overshoot past the static equilibrium state of the system. Use $m = 3$ kg, $c = 18$ N-s/m and $k = 108$ N/m.



Example A2.13

Given: Block A strikes stationary block B with a speed of v. Upon impact, A sticks to B. Assume all surfaces to be smooth.

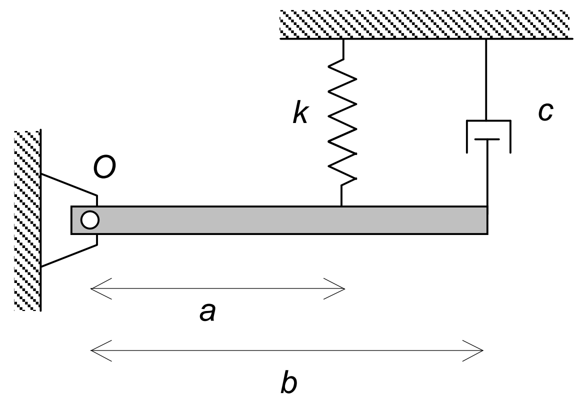
Find: The time history of motion $x(t)$ of the system after A sticks to B. Use $v = 30$ m/s, $k = 3000$ N/m, $c = 30$ N-s/m and $m = 10$ kg.



Example A2.14

Given: The system moves in a horizontal plane.

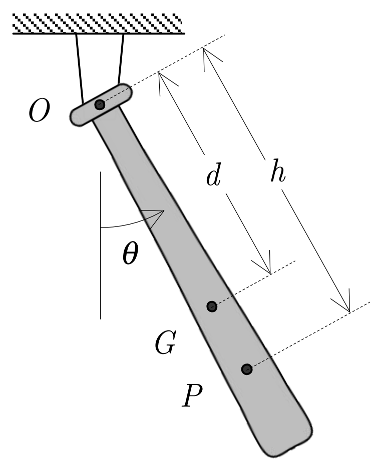
Find: The value of c for critical damping. Assume small oscillations.



Example A2.15

The location center of percussion P for a rigid body is given by: $h = I_O/md$, where h is the distance from the support point O to P, d is the distance from the support point O to the center of mass G and I_O is the mass moment of inertia of the body about the support point O. In this example, we will explore using the free vibration response of a baseball bat suspended from support point O to determine the location of the bat's center of percussion.

1. Draw an FBD of the bat.
2. Develop the equation of motion (EOM) of the bat in terms of the angle θ. Linearize this EOM for small θ (recall that for small θ we have $\sin\theta \approx \theta$).
3. Based on your linearized EOM, what is the natural frequency of free response of the bat in terms of the parameters of the problem?
4. Determine the relationship between the distance h to the center of percussion and the natural frequency of free oscillations for the bat.
5. Discuss how you could set up a simple experiment to determine the location of the center of percussion of the bat.



Example A2.16

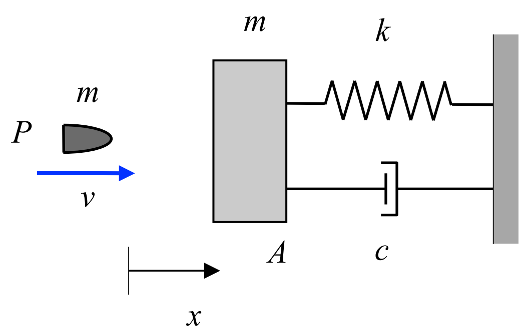
Given: When particle A is at rest and with the spring unstretched, a projectile P traveling with a speed of v impacts and immediately sticks to A.

Find: For this problem:

a) Determine the speed of A immediately after impact. (HINT: Use conservation of momentum for P and A together to determine this speed. Ignore the influence of the spring and dashpot on the motion of the block during impact.)

b) Using the coordinate x, determine the equation of motion for the system for times following the impact of P and A.

c) Determine the response found from the equation of motion in b) above. What is the maximum displacement of A during this response?



Use the following parameters: $v = 10$ m/sec, $m = 4$ kg, $k = 3200$ N/m and $c = 64$ kg/sec.

Example A2.17

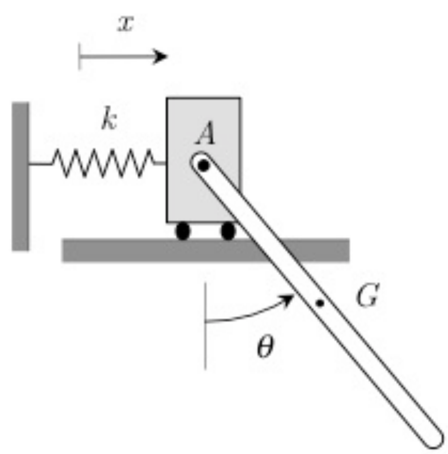
Given: The two-DOF system shown is described by the coordinates x and θ. The block and bar each have a mass of m. The thin bar is homogeneous in its mass distribution and has a length of L. Let $g/L = 2k/m$.

Find: For this problem:

a) Determine the mass and stiffness matrices for the linearized equations of motion for the system corresponding to small motion of the coordinates x and θ.

b) Determine the natural frequencies and modal vectors for the system. Leave your answers for frequencies in terms of m and k and for modal vectors in terms of L.

c) Determine the response of the system for initial conditions of $x(0) = A$, and $\theta(0) = \dot{x}(0) = \dot{\theta}(0) = 0$.



Example A2.18

Given: The absolute coordinates y_1, y_2 and y_3 are used to describe the motion of A, B and the center of mass G of the homogeneous wheel. Blocks A and B, as well as the wheel, each have a mass of m.

Find: For this problem:

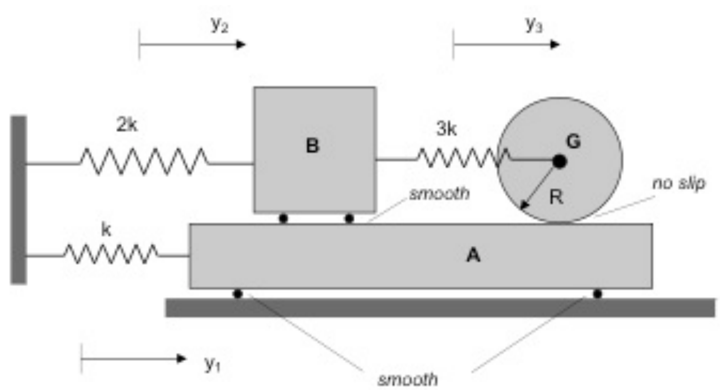
a) Determine the mass and stiffness matrices for the system corresponding to the coordinates y_1, y_2 and y_3.

b) Derive the characteristic equation for the system. Express this characteristic equation in terms of non-dimensional natural frequencies $\omega/\sqrt{k/m}$.

c) Determine the natural frequencies from the characteristic equation found in b). You will need to use a numerical solver from Matlab (or Mathematica). Leave your final answers in terms of m and k.

d) Using your results from c), determine the modal vectors.

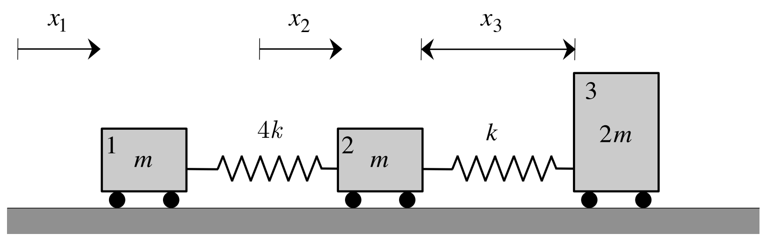
e) Numerically verify the orthogonality properties of the modal vectors: $\vec{Y}^{(i)T}[M]\vec{Y}^{(j)} = \vec{Y}^{(i)T}[K]\vec{Y}^{(j)} = 0;\ i \neq j$.



Example A2.19

Given: The system shown below is released from rest with the initial displacement conditions of $x_1(0) = x_2(0) = 0$ and $x_3(0) = A$.

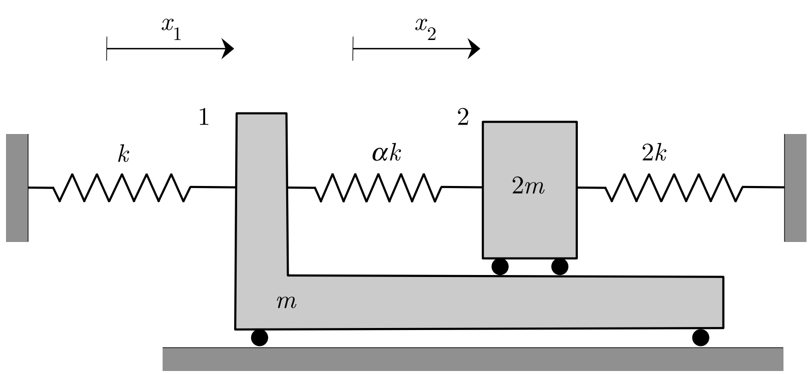
Find: Determine the responses $x_1(t)$, $x_2(t)$ and $x_3(t)$.



Example A2.20

Given: Consider the two-DOF system shown below whose motion is to be described by the absolute generalized coordinates x_1 and x_2.

Find: For this problem:
a) Determine the natural frequencies and modal vectors for the system.
b) Determine the beat period of response for the system corresponding to $\alpha << 1$.



Example A2.21

Given: Consider the damped two-DOF system shown below whose motion is to be described by the absolute generalized coordinates x_1 and x_2.

Find: For this problem:
a) Determine the undamped natural frequencies and modal vectors for the system.
b) Suppose we would like to create a Rayleigh-damped system: $[C] = \alpha[M] + \beta[K]$ where $\alpha = c/m$ and $\beta = 2c/k$. Determine values for c_2, c_2 and c_3 that produces this desired Rayleigh damping. These values should be in terms of the parameter c.
c) Write down the two modally-uncoupled EOMs. What are the two modal damping ratios ζ_1 and ζ_2 corresponding to $c/\sqrt{km} = 0.1$?