Example A3.1

Given: Consider the axial motion of the thin rod shown below. The rod is made up of a material having Young's modulus E and mass density ρ, and has a cross-sectional area of A. The rod has a free end at $x = 0$ and fixed end at $x = L$.

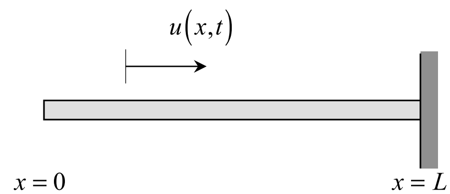
Find: For this problem:

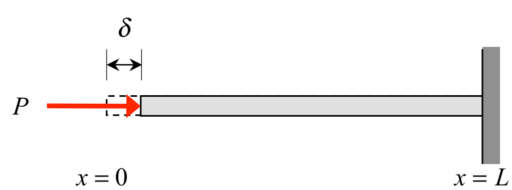
a) Determine the natural frequencies and modal functions for this rod, ω_j and $U^{(j)}(x)$, respectively, for $j = 1, 2, 3, \ldots$.

b) Suppose that an axial load P is applied to the left end of the rod in such a way that the rod is statically deformed with an end deformation of δ. This load is removed quickly with the rod still being at rest. If the subsequent motion of the rod is written as:

$$u(x,t) = \sum_{j=1}^{\infty} (c_j cos\omega_j t + s_j sin\omega_j t)\, U^{(j)}(x)$$

write down expressions for the response coefficients c_j and s_j.



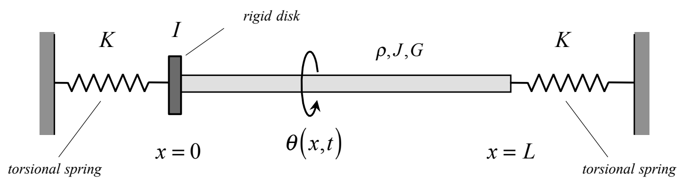


Example A3.2

Given: Consider the three continuous systems shown below.

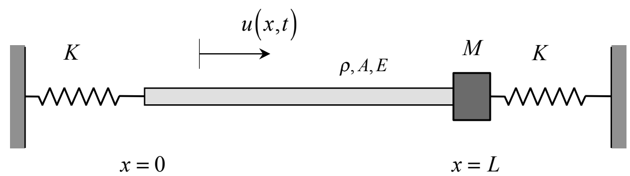
Find: For each system, DERIVE the boundary conditions at both $x = 0$ and $x = L$. Your derivations must include appropriate free body diagrams.

a) torsional motion of shaft



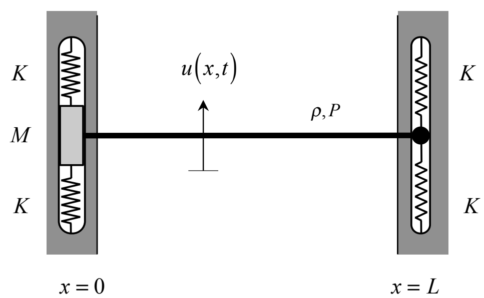
$$\alpha = \frac{KL}{GJ} \qquad \gamma = \frac{I}{\rho JL}$$

b) axial motion of rod



$$\alpha = \frac{KL}{EA} \qquad \gamma = \frac{M}{\rho AL}$$

c) transverse motion of string

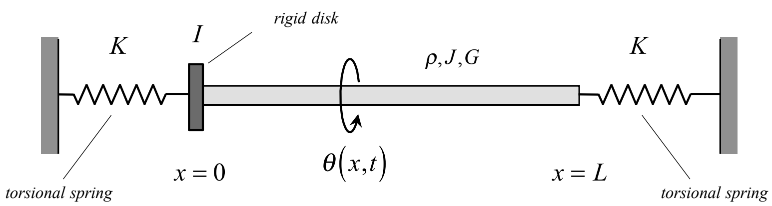


$$\alpha = \frac{KL}{P} \qquad \gamma = \frac{M}{\rho L}$$

Example A3.3

Given: Consider the shaft shown below.

Find: For this problem:
a) Develop the characteristic equation (CE).
b) Make a hand sketch of the terms in the CE showing the locations of the roots of the CE.
c) Based on your sketch above, place lower and upper bounds on the first four natural frequencies for the system.
d) Using Matlab, or an equivalent application, determine numerical values for the first four natural frequencies, as well as the corresponding modal functions. Use $\alpha = 2$ and $\gamma = 1$ in your numerics. Your answers should be in terms of system parameters indicated in the figure.



$$\alpha = \frac{KL}{GJ} \qquad \gamma = \frac{I}{\rho JL}$$

Example A3.4

Given: Consider the transverse motion of the thin beam shown below.

Find: For this problem:
a) Develop the characteristic equation (CE).
b) If a closed-from solution of the CE is possible, solve it. If not, then: i) make a hand sketch of the terms in the CE showing the locations of the roots of the CE; ii) place lower and upper bounds on the first four natural frequencies for the system; iii) using Matlab, or an equivalent application, determine numerical values for the first four natural frequencies, as well as the corresponding modal functions. Your answers should all be in terms of system parameters such as E, I, A, L and ρ.
c) Make a hand sketch of first four modal functions.