## Chapter 2

## Planar Rigid Body Kinematics Homework

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## Homework H.2.A

Given: A square plate (having side lengths of $b=\sqrt{2} \mathrm{~m}$ ) rotates with a counterclockwise sense at a rate of $\Omega=5 \mathrm{rad} / \mathrm{s}$ about a shaft passing through corner O . At the position shown below, corner $B$ is directly above the shaft $O$.

Find: Consider the following two parts of this problem:
(a) For the first part, we are given that the rotation rate of the plate is changing at a rate of $\dot{\Omega}=$ $10 \mathrm{rad} / \mathrm{s}^{2}$. Determine the velocity and acceleration vectors for corners A and B of the plate. Make sketches of these vectors.
(b) For the second part, we are not given information on $\dot{\Omega}$. Instead, we know the acceleration of corner A to be in the negative y-direction (the x -component is zero), as shown in the figure below. For this, determine the numerical value of $\dot{\Omega}$ and of the acceleration vector for corner B. Make a sketch of the acceleration vector for corner B.


Part (a)


## Part (b)

## Homework H.2.B

Given: A disk, having an outer radius of $R$, rotates with a rate of $\Omega$ about a shaft passing through its center O , with $\Omega$ increasing at a rate of $\dot{\Omega}$. The shaft is supported by a pair of bearings on cart A. Cart A is moving to the right with a speed of $v_{A}$ and an acceleration of $a_{A}$. At the instant of interest, point B on the perimeter of the disk is at the same height as O .

Find: For this problem:
(a) Determine the velocity of point B. Write your answer as a vector in terms of its $x y z$ components.
(b) Determine the acceleration of point B. Write your answer as a vector in terms of its $x y z$ components.


SIDE view


RIGHT END view of disk

## Homework H.2.C

Given: Rigid body AB is shaped as quarter-circle arc with a radius of $R$. End B of the bar is constrained to move along a vertical wall, whereas end $A$ moves along an incline at an angle of $\theta=$ $53.13^{\circ}$ with respect to the horizontal. At the instant shown, the center O of the AB arc is directly below end B , and end A moves with a constant speed of $v_{A}$.

Find: For this problem:
(a) Determine the velocity and acceleration of end B of the bar. Express your answers as vectors and in terms of the parameters of $v_{A}$ and $R$.
(b) Is the speed of B increasing, decreasing or constant?


## Homework H.2.D

Given: Roller D of the mechanism shown is moving downward along a straight vertical surface with a constant speed of $v_{D}$. At the instant shown, link AB is vertical.

Find: For this position:
(a) Determine the angular velocities of links AB and BD . Write your answers as vectors.
(b) Determine the angular accelerations of links AB and BD . Write your answers as vectors.


Use the following parameters in your analysis: $\theta=53.13^{\circ}, L=2 \mathrm{~m}$ and $v_{D}=15 \mathrm{~m} / \mathrm{s}$.

## Homework H.2.E

Given: The compound wheel assembly shown below is driven by a cable attached to the outer rim of the assembly at point $A$. The wheel rolls without slip at point $B$ with point $C$ moving to the right with a speed of $v_{C}$, and the acceleration at point A is given by $\vec{a}=a_{x} \hat{i}+a_{y} \hat{j}$

Find: Determine the acceleration of the center point C of the pulley.


Use the following parameters in your analysis: $R=0.1 \mathrm{~m}, r=0.0125 \mathrm{~m}, \phi=0^{\circ}, a_{x}=8 \mathrm{~m} / \mathrm{s}^{2}$ and $a_{y}=-3 \mathrm{~m} / \mathrm{s}^{2}$.

## Homework H.2.F

Given: The circular disk shown rolls without slipping on a straight horizontal surface. Bar AB is pinned to point A on the disk, with end B constrained to move along a smooth horizontal surface with a constant speed $v_{B}$. At the position shown, A is directly to the right of the center O of the disk.

Find: For this position:
(a) Determine the angular velocities of link AB and of the disk. Write your answers as vectors.
(b) Determine the angular accelerations of link AB and of the disk. Write your answers as vectors.


Use the following parameters in your analysis: $R=6 \mathrm{in}, L=10 \mathrm{in}$ and $v_{B}=100 \mathrm{in} / \mathrm{s}$.

## Homework H.2.G

Given: The mechanism shown is made up of rigid links $\mathrm{OA}, \mathrm{AB}$ and CD. Link OA has a pin joint at end O and is known to be rotating in the clockwise sense about O with a rotation rate of $\omega_{O A}$. AB is pinned to OA at end A and is pinned to a slider at B , with B moving along a horizontal guide. Link CD connects the center C of AB to a second slider at D through pin joints, with this slider constrained to move along a vertical guide. At the instant shown link OA is oriented vertically, B is directly below D and D is directly to the right of A .

Find: For this problem:
(a) Locate the instant center (IC) for link AB. Based on the location of this IC, what is the speed of pin C and the direction of travel for C , as well as the speed of slider B ?
(b) Locate the IC for link CD. Based on this location, determine the speed of slider D.

NOTE: Please use only the instant center approach for this problem. Do not use vector analysis to find your answers.


## Homework H.2.H

This problem has three parts. In each part, you are asked to use the instant center approach in answering the questions related to the problems. In all cases, the figures are drawn to scale. Please use a straight edge when making your drawings.

PART A
In the mechanisms shown below, link OA is rotating in the counterclockwise sense. For the position shown of EACH mechanism:
(a) Determine the location of the instant center for link AB .
(b) Determine the directions of rotation for links AB and BD. Justify your answers in words.
(c) Which is larger: $\left|\omega_{O A}\right|$ or $\left|\omega_{A B}\right|$ ? Justify your answers in words.


PART B
In the mechanism shown below, link OA is rotating in the counterclockwise sense.
(a) Determine the locations of the instant centers for links $\mathrm{AB}, \mathrm{BE}$ and EG.
(b) Determine the directions of rotation for links AB, BE and EG. Justify your answers in words.


## PART C

Link AB, having a length of $L=5 \mathrm{in}$, is part of a planar mechanism. At the instant shown, the velocities of points $A$ and $B$ are known to be both perpendicular to a line connecting $A$ and $B$, with $v_{B}=3 v_{A}=30 \mathrm{in} / \mathrm{s}$. Determine the location of the instant center for link AB.


## Homework H.2.I

Given: A mechanism is made up of rigid links $\mathrm{OA}, \mathrm{AB}, \mathrm{BC}$ and DE . A slider is pinned to end E of link DE and is constrained to move along a horizontal guide. For the position shown, link OA is rotating in the counterclockwise sense about O with a constant rotation rate of $\omega_{O A}$, with links OA and BC being vertically oriented and link AB being horizontally oriented.

Find: For the position shown:
(a) Use the instant center approach to determine the angular velocities of links $\mathrm{AB}, \mathrm{BC}$ and DE , along with the speed of slider E.
(b) Use vector analysis to determine the angular accelerations of links $\mathrm{AB}, \mathrm{BC}$ and DE , along with the acceleration of slider E. Is the speed of E increasing, decreasing or constant at this instant?


## Homework H.2.J

Given: A mechanism is made up of block B , a circular disk having an outer radius of $R$ and link AD . Block B is constrained to move along a horizontal surface with the disk being able to roll without slipping on block B . Link AD is pinned to point A on the disk and to a slider at D , with the slider being constrained to move along a horizontal guide. At the instant shown, A is directly to the right of the center C of the disk, and block B moves to the left with a constant speed of $v_{B}$.

Find: For the position shown:
(a) Determine the angular velocities of the disk and link AD. Write your answers as vectors.
(b) Determine the angular accelerations of the disk and link AD. Write your answers as vectors.


