Chapter 1

Particle Kinematics Homework

Problem H.1.A

Given: A particle P travels on a path described by the Cartesian coordinates of $y = cx(b - x)$, where x and y have the units of meters. The x-component of velocity, $\dot{x}$, for P is constant.

Find: For this problem:

(a) Make a sketch of the path of P over the range of $0 < x < b$.

(b) Determine the Cartesian components of the velocity and acceleration of P at $x = 0$. Add a sketch of the velocity and acceleration vectors for P to your path drawn above.

(c) Determine the Cartesian components of the velocity and acceleration of P at $x = b/2$. Add a sketch of the velocity and acceleration vectors for P to your path drawn above.

Use the following parameters in your analysis: $b = 2$ m, $c = 5$/m and $\dot{x} = 4$ m/s.

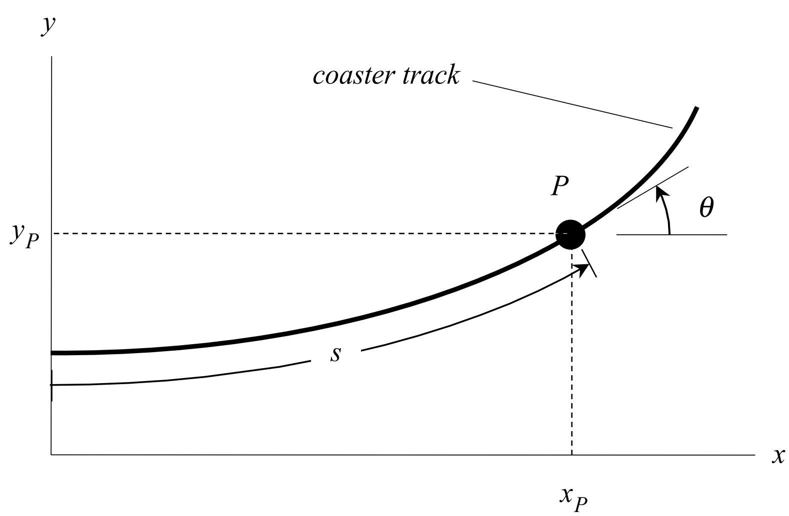
Homework H.1.B

Given: A propelled cart P travels on a rollercoaster track. Let s represent the distance traveled by P on this track, where s has units of feet. In terms of the distance s, the radius of curvature of the track, the angle of the tangent to the track and the speed of P are known to be $\rho(s) = 1/bs$, $\theta(s) = bs^2/2$ and $v(s) = d - cs^2$, respectively.

Find: For this problem:

(a) Determine the path variable components of velocity and acceleration of P as a function of s.

(b) Evaluate your results in (a) above for $s = 100$ ft. Make sketches of the velocity and acceleration vectors at this position, giving special attention to the orientation of these two vectors in your sketches.



Use the following parameters in your analysis: $b = 1 \times 10^{-4}$ /ft^2, $d = 100$ ft/s and $c = 1 \times 10^{-3}$/ft $\cdot$ s.

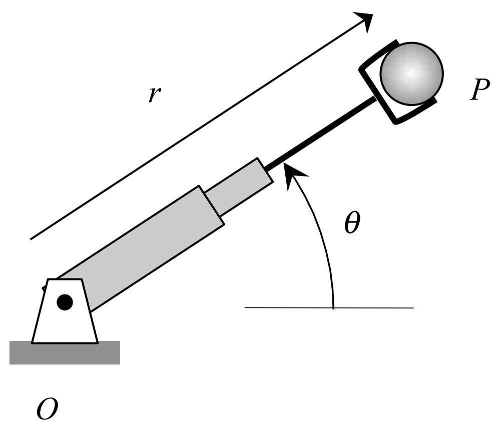
Problem H.1.C

Given: A rotating and telescoping robotic arm is gripping a small sphere P in its end effector. The arm is rotating counterclockwise with a constant angular speed of $\dot{\theta}$. The arm is extending such that the radial distance from O to P is related to the rotation angle θ by the following equation:

$$r(\theta) = R_0 + R_1 \cos 2\theta$$

where r and θ are given in terms of meters and radians, respectively.

Find: Determine the velocity and acceleration of the sphere P. Write your answers as vectors in terms of the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$.

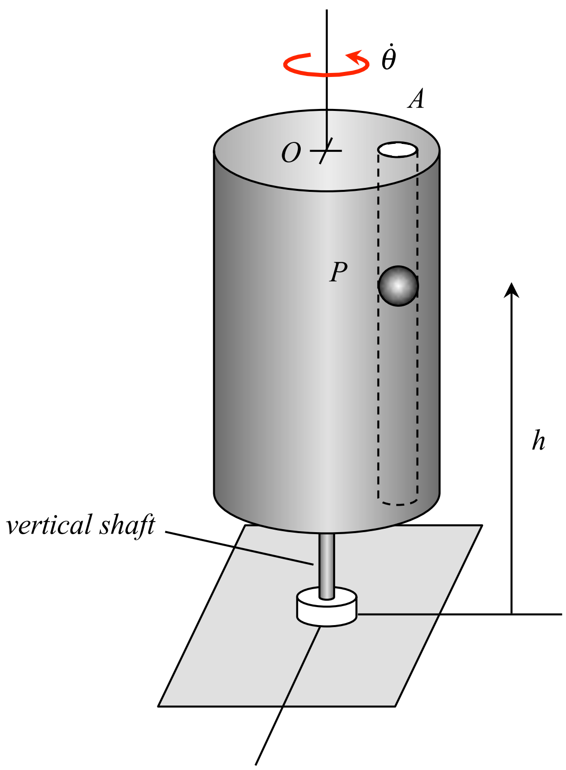


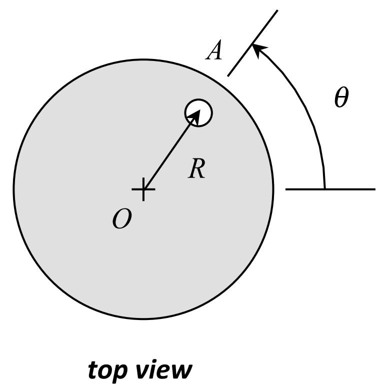
Use the following parameters in your analysis: $R_0 = 2$ m, $R_1 = 0.5$ m, $\theta = \pi/2$ rad and $\dot{\theta} = 2$ rad/s.

Homework H.1.D

Given: A cylinder rotates about a vertical shaft with a non-constant rate of $\dot{\theta}$, where $\dot{\theta}(0) = 0$. At time $t = 0$, particle P is dropped from rest into a smooth, vertical tube that is cut into the cylinder, with $h = h_0$. NOTE: As particle P drops through the tube, it has a vertical component of acceleration of -g, where g is the gravitational constant.

Find: At the instant when $h = h_0/2$, determine the speed of P and the magnitude of acceleration of P.





Use the following parameters in your analysis: $\ddot{\theta} = 2$ rad/s^2 = constant, $h_0 = 10$ ft and $R = 0.5$ ft.

Homework H.1.E

Given: A particle P travels in the x-y plane with a path whose coordinates are given as a function of time t as: $x(t) = 16 - 12t$ and $y(t) = 2 + 15t - 3t^2$, where x and y are in meters, and t is in seconds.

Find: For this problem:

(a) Determine the velocity and acceleration of P in terms of their x-y components.

(b) Make a sketch of the velocity and acceleration vectors for P.

(c) Determine the rate of change of speed of P and the radius of curvature for the path of P.

Use the following parameters in your analysis: $t = 10$ s.

Problem H.1.F

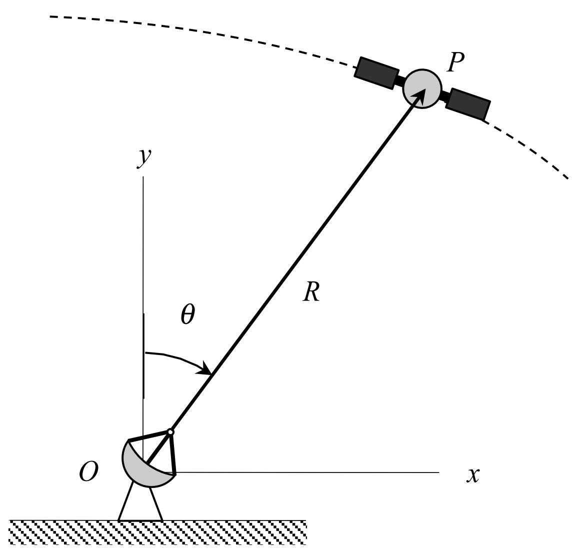
Given: A radar observatory at point O on the surface of the earth is tracking a satellite P by measuring R and θ, and their time dervatives. Neglect any effects due the earth's rotation in the following analysis.

Find: For this problem:

(a) Determine the polar components of the velocity of P, with O being the origin of the polar coordinate system.

(b) Make a sketch of P in this position, including a sketch of the velocity vector for P.

(c) Determine the x and y components of the velocity of P.



Use the following parameter in your analysis: $\theta = 0$, $R = 8{\times}10^4$ m, $\dot{R} = 2{\times}10^4$ m/s, and $\dot{\theta} = 0.4$ rad/s.

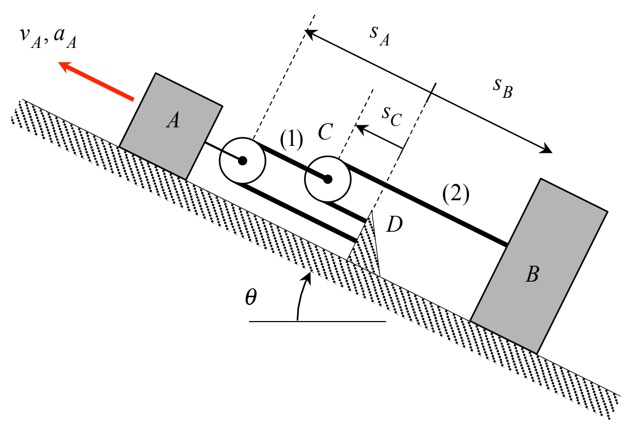
Problem H.1.G

Given: An inextensible cable (1) is attached to fixed ground at end D and to the center of pulley C at the other end, with the cable wrapped around a pulley that is mounted on block A. A second inextensible cable (2) is attached to fixed ground at end D and to block B, with the cable being wrapped around pulley C. Block A is known to have a speed and acceleration of v_A and a_A, respectively, up the incline on which A and B move.

Find: For this problem:

(a) Determine the speed of B

(b) Determine the magnitude of the acceleration of B.



Use the following parameters in your analysis: v_A= 10 m/s, a_A = 4 m/s^2 and θ = 24°.

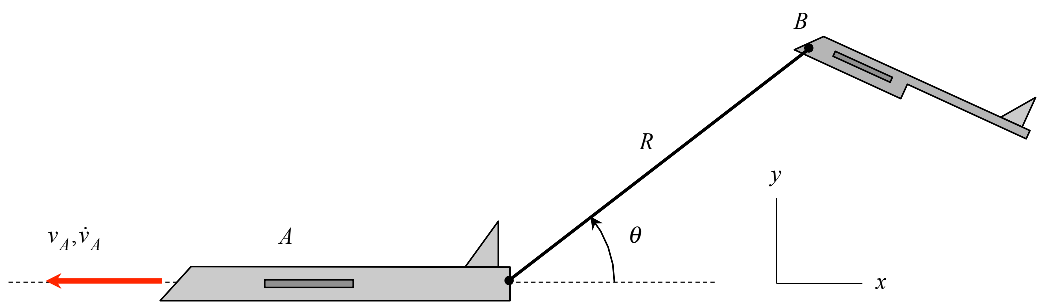
Problem H.1.H

Given: Aircraft A is traveling along a straight-line path with a speed of v_A that is increasing by an amount of $\dot{v}_A$. The aircraft is towing a glider B with a cable that has a length of R. The angle θ of the towline is increasing by a constant amount of $\dot{\theta}$.

Find: For this problem:

(a) Determine the velocity vector of the point on glider B to which the cable is attached.

(b) Determine the acceleration vector of the point on glider B to which the cable is attached.



Use the following parameters in your analysis: $R = 80$ m, $v_A = 100$ m/s, $\dot{v}_A = 4$ m/s^2 and $\theta = 20°$ and $\dot{\theta} = 0.1$ rad/s.

