

USDOT Region V Regional University Transportation Center Final Report

NEXTRANS Project No. 034OY02

**OPTIMAL CONDITION SAMPLING FOR A NETWORK OF  
INFRASTRUCTURE FACILITIES**

By

Rabi G. Mishalani, Principal Investigator  
Associate Professor of Civil and Environmental Engineering  
and Geodetic Science  
The Ohio State University  
mishalani@osu.edu

and

Prem K. Goel, Co-Principal Investigator  
Professor of Statistics  
The Ohio State University  
goel.1@osu.edu



## **DISCLAIMER**

Partial funding for this research was provided by the NEXTRANS Center, Purdue University under Grant No. DTRT07-G-005 of the U.S. Department of Transportation, Research and Innovative Technology Administration (RITA), University Transportation Centers Program. Additional funding was provided by The Ohio State University (OSU) including the College of Engineering and the Department of Statistics. The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the Department of Transportation, University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.



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# TECHNICAL SUMMARY

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Final Report, December 31, 2011

## Optimal Condition Sampling for a Network of Infrastructure Facilities

### Introduction

In response to the developments in inspection technologies, infrastructure decision-making methods evolved whereby the optimum combination of inspection decisions on the one hand and maintenance and rehabilitation decisions on the other are determined based on an economic evaluation that captures the long term costs and benefits. Recently, sample size has been included in inspection, maintenance, and rehabilitation (IM&R) decision-making as a decision variable when considering a single facility. While, the question of dealing with a network of facilities in making IM&R decisions has been addressed in the literature, this treatment does not consider condition sampling whereby each facility could require a different set of sample sizes over time.

This report presents an overview of the methodology developed to address the network level problem whereby the uncertainty due to condition sampling is captured and its related decision variables included in the IM&R decision making process. An example application is described and results and insights are presented. The parameters of the example of interest are determined by drawing upon various cases reported in the literature to arrive at a realistic base scenario for analysis. A comprehensive sensitivity analysis is also conducted to explore the effect of various factors on the optimal solution.

### Findings

Based on examining the base scenario, it is clear that larger sample sizes can compensate for decreasing inspection accuracy up to a point where the degrading accuracy is so large, increasing the sample sizes does not offer much if any value. In addition, and not surprisingly, a stricter annual budget constraint will result in reduced expected IM&R cost and larger expected user and terminal costs. And, overall, an increase in the expected total cost is expected. Moreover, the effect of including sampling as a decision variable is found to be appreciable in terms of the expected total cost at optimality.

Based on examining all scenarios combined, the sensitivity analysis revealed that the user cost, annual budget constraint, terminal cost, and the spatial correlation function have appreciable impact on the optimal solution. Among these four factors, the impacts of the user cost and annual budget constraint are the most marked. Furthermore, these factors do interact with one another and the most notable

interaction in terms of its magnitude and implications to agencies is that between user cost and the budget constraint.

## Recommendations

It is important to conduct a more extensive evaluation to quantify the value of capturing sampling uncertainty and including sampling as a decision variable. Such a comprehensive evaluation has been conducted in the literature for the facility level problem in the absence of a budget constraint. However, it remains to be undertaken as part of future research for the network level problem.

Clearly, the results are limited given the hypothetical, albeit realistic, nature of the scenarios considered in the numerical analyses presented in this report. Therefore, it is critical to demonstrate and assess the value of the developed methodology under field conditions to achieve a more comprehensive and realistic evaluation and possible refinements.

Another important limitation of the developed framework worth addressing is the absence of capturing facility interactions. Important interactions have been captured in the literature. However, condition sampling is not considered in such investigations. Developing a decision-making framework that simultaneously captures facility interactions and includes condition sample sizes across facilities and over time as decision variables would be worthwhile.

## Contacts

*For more information:*

### **Rabi G. Mishalani**

Principal Investigator  
Dept. of Civil & Environ. Engineering and Geodetic Science  
The Ohio State University  
mishalani.1@osu.edu

### **Prem Goel**

Co-Principal Investigator  
Department of Statistics  
The Ohio State University  
goel.1@osu.edu

### **NEXTRANS Center**

Purdue University - Discovery Park  
2700 Kent B-100  
West Lafayette, IN 47906

nextrans@purdue.edu  
(765) 496-9729  
(765) 807-3123 Fax

[www.purdue.edu/dp/nextrans](http://www.purdue.edu/dp/nextrans)

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Professor of Statistics  
The Ohio State University  
goel.1@osu.edu

Report Submission Date: December 31, 2011

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## 1 INTRODUCTION

Transportation infrastructure systems consist of spatially extensive and long-lived sets of facilities. Over the past two decades, several new non-destructive inspection technologies have been developed and applied in collecting raw condition data and processing them to produce useful condition input to infrastructure inspection, maintenance, and rehabilitation (IM&R) decision-making aimed at minimizing total expected life-cycle cost. Inspection deals with the gathering of data on the extent of facility damage. The data may be collected by visual inspection, through manual measurements, or by automated sensors. An average of collected damage measurements over a facility (defined as a homogeneous section) is an estimate of the current condition of that facility and, in turn, is one primary input to maintenance and rehabilitation (M&R) decision-making.

The developments in nondestructive inspection technologies make it possible to estimate facilities' conditions using large quantities of data. The quality of measurements, the sample size, and the nature of correlation among condition variables at different locations determine the accuracy of condition estimates. Naturally, more accurate estimates have the potential to lead to more effective maintenance and rehabilitation decisions. Consequently, the expected combined user costs and maintenance and rehabilitation costs are reduced over the planning horizon. However, more accurate information requires more resources such as increased inspection frequency, advanced inspection sensor technologies, larger sample sizes, or possibly less correlated observations, as well as data processing methods that appropriately combine all this information.

In response to the developments in inspection technologies, decision-making methods evolved whereby the optimum combination of inspection decisions on the one hand and M&R decisions on the other are determined based on an economic evaluation that captures the long term costs and benefits. Madanat (1993), Madanat & Ben-Akiva (1994), and Ellis et al. (1995) extended the Markov Decision Process (MDP) based infrastructure management decision-making framework (Golabi et al. 1982, Carnahan et al. 1987), which captures forecasting uncertainty, to the Latent Markov Decision Process (LMDP) framework by incorporating measurement errors associated with condition inspection. In addition, inspection technology and timing were introduced as decision variables. Recently, the LMDP framework was extended to include condition sample size as a decision variable in IM&R decision-making (Mishalani & Gong 2009). However, several of the aforementioned studies including this latest extension only considered decisions for a single facility.

The question of dealing with a network of facilities in making M&R decisions has been addressed in the literature through a variety of formulations – *e.g.* Golabi et al. (1982), Golabi & Shepard (1997), Murakami & Turnquist (1995), Smilowitz & Madanat (2000), Durango-Cohen and Sarutipand (2007), and Kuhn (2010). However, the treatments, while addressing several important issues, do not consider condition sampling whereby each facility could require a different and time-varying sample sizes. Doing so optimally is valuable given the network nature



of facilities that most infrastructure agencies are responsible for, the increasing number of inspection technology choices with possible varying degrees of accuracy and cost, and budget constraints agencies have to work within.

In the next section, a methodology for IM&R decision-making at the network level is introduced followed by the presentation of a new extension taking into account recent developments addressing sampling at the single facility level. The result is a methodology that captures the uncertainty due to condition sampling and includes sampling as decision variables in the IM&R decision-making process at the network level. In the third section, a numerical application of the methodology and a sensitivity analysis based on a realistic literature- and practice-derived example network of facilities are discussed and insights regarding condition sampling at the network level are derived. The final section summarizes the study and its findings, and presents directions for future research.

## **2 NETWORK-LEVEL APPROACH AND DEVELOPED METHOD**

In this section, the network level approach based on a randomized policy is first summarized. Based on this approach the developed methodology is presented. In doing so, the treatment of sampling at the facility level is incorporated in solving the problem at the network level.

### **2.1 Network-level approach without sampling**

Some network-level IM&R decision-making methods in the literature adopted randomized policies (Golabi 1982, Golabi & Shepard 1997, Murakami & Turnquist 1995, Smilowitz & Madanat 2000, Harper et al. 1990, Gopal & Majidzadeh 1991). Smilowitz & Madanat (2000) proposed a linear programming formulation for solving the infrastructure IM&R optimization problem at the network level considering inspection technology and timing as decision variables. Given the advances that study achieved in capturing these inspection decisions, it provides a natural basis for the methodology presented in this report. Before describing the formulation proposed by Smilowitz & Madanat (2000), certain critical elements are first introduced.

Assessed facility condition is assumed to fall into one of a finite number of discrete condition states. Considering that inspection is not perfect, it is assumed that the measurement of condition states does not produce the true condition state. In order to infer the true condition states based on measurements, the nature of measurement error has to be considered. To do so, the concept of the information vector is introduced. This vector is a probability mass function on all possible condition states conditional on prior information. This prior information consists of the initial condition state before any decisions are made (*i.e.*, at time 0), the M&R actions applied up to the current point in time, and all the condition measurements taken including the most recent inspection.

In the formulation developed by Smilowitz & Madanat (2000), two types of actions are considered. One represents the M&R actions to be performed and the other represents whether to inspect condition or not. The various M&R actions have different costs and result in different

condition state transition probabilities over time. The inspection action space includes taking a condition measurement or not. Inspection improves the understanding of the true condition state. That is, the information vector will be more concentrated around the true condition state.

Transition probabilities specify how facility condition evolves during the next time period, given the current condition state, age, and M&R action applied. More specifically, a transition probability represents the likelihood of a facility to transition to a certain condition state in one period given the condition state it is currently in. Therefore, transition probabilities can be organized into a matrix representing all combinations of transitions from state to state. Two facilities in the same condition state to which the same M&R action is applied will have different transition probabilities if their age is different. Age is defined as the number of years since the most recent rehabilitation action was applied. The facility with lower age has a smaller probability of deteriorating to a poorer state during the next period. Therefore, it is important to allow for non-stationary transition probabilities conditional on the age of a facility.

Finally, two cost components are considered. The first component is the cost incurred due to IM&R and depends on the specific actions performed. The other component is the user cost which only depends on the condition state.

Given the above elements, the problem of determining the best decision can then be formulated as a linear program with the following components.

1. *Decision variables*:  $W_t(I,y,a,r)$  denotes the number of facilities at time  $t$  whose information vector is  $I$  and age is  $y$  on which M&R action  $a$  will be performed, and which will be inspected if  $r = 1$  and not if  $r = 0$ . The nature of the decision variables when solving the network problem is that of a randomized policy. That is, facilities associated with the same information vector could receive different actions (optimally determined). Such a result is possible due to the presence of budget and condition constraints (discussed in more detail below). It is important to note that the specific facilities associated with the same information vector that are to receive one set or another of the determined optimal actions is not part of the solution outcome. Such decisions could be arrived at by managers at a subsequent stage given the optimal randomized policy.
2. *Objective function*: For each feasible realization of the decision variables, the expected total discounted cost includes the user and IM&R costs and is a linear function of the decision variables.
3. *Constraints*: (i) Non-negativity constraints guarantee that each decision variable is non-negative. (ii) Conservation constraints ensure the conservation of facilities over time. That is, the information vectors must transition from one period to the next in a manner consistent with the condition state transition probabilities. (iii) The initial distribution of facilities as defined by a set of information vectors is assumed known and takes the form of an initial state constraint. (iv) Condition state constraints require that the proportion or number of

facilities in the condition states considered to be poor is bounded by a maximum value each year. (v) Budget constraints require that the IM&R cost is bounded by a maximum and possibly a minimum value each year.

At this point, the problem is reduced to finding the values of the decision variables that satisfy all the constraints and achieve the minimal objective function value. Since the objective function and all the constraints are linear with respect to the decision variables, the problem can be solved using linear programming.

## **2.2 Developed network-level approach incorporating sampling**

The formulation developed by Smilowitz & Madanat (2000) described above generalizes the facility level framework developed by Madanat & Ben-Akiva (1994) to a network level one by introducing randomized decisions and solving a linear programming problem. It includes many realistic elements. However, an important decision variable and an associated modeling element are not captured in this formulation, namely, the sample size and spatial correlation among measurements of condition taken at different locations along the same facility.

The role of sampling is to increase the accuracy of the information regarding facility condition state. On the one hand, more samples result in higher accuracy. On the other hand, more samples will introduce more cost. Thus, sample sizes for each facility over time are important decision variables. Effectively, in the formulation developed by Smilowitz & Madanat (2000), for a facility either one sample is taken or not, which is quite limiting. Therefore, the extended formulation developed in this report includes sample sizes as decision variables. In addition, in the application two condition inspection technologies are considered rendering the set of inspection decisions more flexible. Once multiple samples are taken from the same facility, the spatial correlation among these condition observations must be considered in quantifying the combined measurement and sampling uncertainty. Therefore, a spatial correction function (Mishalani & Koutsopoulos 2002) is adopted in determining this uncertainty.

Elements similar to those of the formulation discussed above are first noted. The assessed facility condition is assumed to fall into one of a finite number of discrete condition states. Given the current condition assessment through measurement and sampling along with historical information including previous measurements and IM&R actions, the posterior probability mass function of the true condition state of each facility (*i.e.*, the information vector) can be determined. Also, it is important to keep track of age in addition to time because the transition probabilities (which take the same form discussed above) depend on age.

The variance of the assessed condition is a critical element of the new formulation and constitutes a major departure from the formulation developed by Smilowitz & Madanat (2000). This variance is determined as a function of the measurement technology, sample size, and the characteristics of the facility in terms of its intrinsic variability in condition and the spatial correlation. The determination of this variance is based on the formulation developed by

Mishalani & Gong (2009) for a single facility. Another important departure from the formulation discussed above is the introduction of facility length,  $h$ , in representing the network. This variable influences the value of the determined variance of the assessed condition depending on the sample size, intrinsic variability, and spatial correlation. Finally, in addition to the user and IM&R costs discussed above, the hypothetical terminal cost incurred at the end of the time horizon represents the cost of bringing the facility back to the best condition state for the purpose of equalizing the service life from that point onward.

In light of the elements described above, the problem of determining the best decision can similarly be formulated as a linear program with the following components.

1. *Decision variables*:  $W_t(I, h, y, a, r, n)$  denotes the number of facilities at time  $t$  whose information vector is  $I$ , its length is  $h$ , and of age  $y$  on which M&R action  $a$  will be performed, and inspection technology  $r$  will be used with  $n$  samples. As in the case of the formulation developed by Smilowitz & Madanat (2000), the nature of the decision variables when solving the network problem is that of a randomized policy. That is, facilities associated with the same information vector could receive different actions (optimally determined). Again, such a result is possible due to the presence of budget and condition constraints. And, as in the case of the study by Smilowitz & Madanat (2000), the optimal set of actions specific facilities associated with the same information vector are to receive is not part of the solution outcome.
2. *Objective function*: For each feasible realization of the decision variables, the expected total discounted cost includes the user, IM&R, and terminal costs. Of course, the decision variables now include the sample size  $n$  and the original measurement uncertainty is extended to include measurement and sampling uncertainty taking into account the sample size, facility length, intrinsic variability, and spatial correlation. The objective function remains a linear function of the decision variables.
3. *Constraints*: The constraints are similar to those of the formulation developed by Smilowitz & Madanat (2000) discuss above.

As in the case of the study by Smilowitz & Madanat (2000), the problem is reduced to finding the values of the decision variables that satisfy all the constraints and achieve the minimal objective function value, which can be solved using linear programming.

### 3 NUMERICAL ANALYSES

In this section an example application is described and results and insights are presented. First, the example base scenario is specified. Second, the effect of measurement error is explored under this base scenario. Third, the effect of the annual budget constraint is investigated under the same scenario. Fourth, the value of including sampling as a decision variable is investigated, again under the same scenario. Finally, a set of additional scenarios is specified and a sensitivity

analysis is conducted. These investigations lead to theoretical and practical implications, which are discussed as well.

In all the applications presented in this section, a ten-year planning horizon is considered and each facility in the network is assumed to be in perfect condition at the beginning of the planning horizon. In applying the developed methodology for all the analyses presented in this section, the computational cost associated with the size of the problem must be addressed. This is done through discretizing and grouping information vectors where the resolution of the vector space is set to 0.1.

### 3.1 Base scenario development

The parameters of the example of interest are determined by drawing upon various cases reported in the literature to arrive at a realistic base scenario for analysis. The specification is for the most part based on a realistic example developed by Gong (2006). The parameter values are shown in Table 1.

TABLE 1 Base Scenario Parameter Values (all costs in \$/m<sup>2</sup>)

Condition State	4	3	2	1
Routine maint. cost	0.34	1.63	4.4	14.79
User cost	18.47	56.735	75.155	113.43
Terminal cost	2.675	22.485	55.695	64.87
Rehab. cost	64.87			
Additional user cost due to M&R	1.45 for routine maintenance and 10.13 for rehabilitation			
Additional user cost due to inspection	0.09 for inspection tech. 1 and 0.0015 for inspection tech. 2			
Fixed inspection cost	0.0119 for inspection tech. 1 and 0.0093 for inspection tech. 2			
Unit inspection cost	0.00023 for inspection tech. 1 and 0.000085 for insp. Tech. 2			
Intrinsic variance	Function of the true condition state			
Var. of insp. tech.	Tech. 1: 7.99, tech. 2: 17.95			

### 3.2 Effect of measurement error

To test the effect of measurement error, an important element of the formulation especially in light of sampling, different pairs of values of the standard deviations of the two measurement technologies are specified as shown in Table 2 reflecting a wide spectrum ranging from perfect measurements to very poor measurements. Level 2 represents the base scenario while levels 1 and 9 represent the two extreme scenarios of perfect and poor measurements, respectively.

TABLE 2 Standard Deviations of Measurement Errors

Level	Standard Deviation	
	Tech. 1	Tech. 2
1	0	0
2 (base scenario)	7.99	17.95
3	15	30
4	30	60
5	50	100
6	80	160
7	150	300
8	500	1000
9	1000	2000

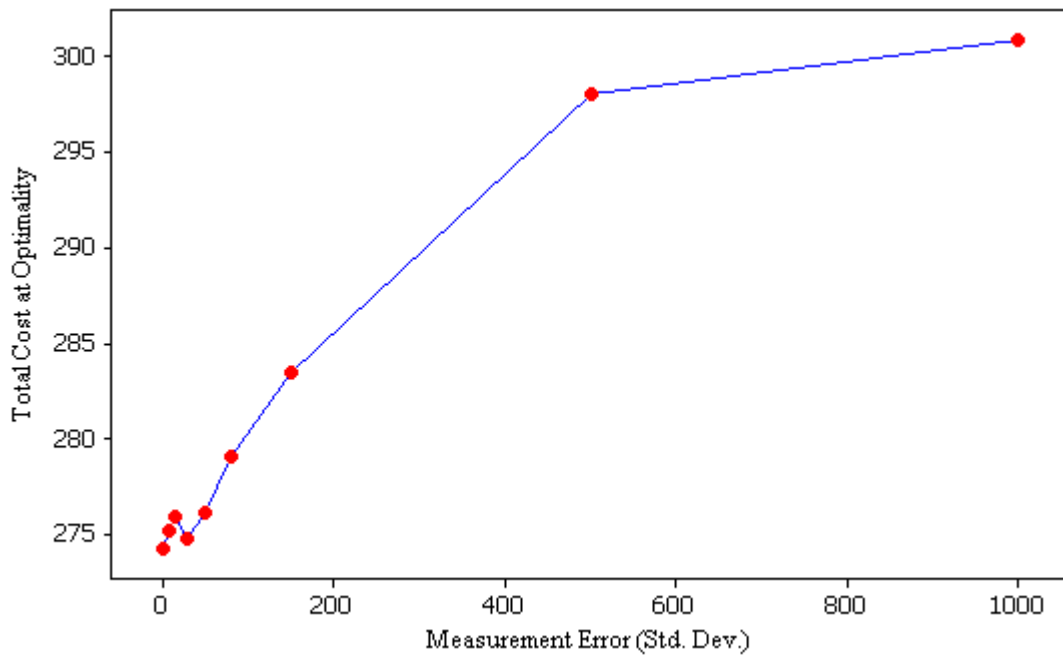


FIGURE 1 Expected total cost versus standard deviation of technology 1’s measurement error.

Figure 1 shows the expected total cost (in  $\$/m^2$ ) at optimality as a function of the standard deviation of measurement technology (of technology 1 in this case). As expected, notice that the total expected cost, in general, increases as the measurement error increases. However, at level 4,

there is a decrease in the expected total cost with respect to levels 2 and 3. This drop is likely due to the discretization and grouping of information vectors. (A higher resolution would naturally produce more accurate results that would avoid such anomalies. However, a higher resolution requires a higher computational cost, which was not practical given the large number of scenarios considered subsequently).

Figure 2 shows the expected inspection cost (in  $\$/m^2$ ) at optimality as a function of the standard deviation of measurement technology (of technology 1 in this case). Notice that the inspection cost, in general, increases as the measurement error increases reflecting the situation where larger samples are taken to compensate for the deteriorating measurement accuracy. At some point, however, the inspection cost starts to decrease reflecting the loss of value of information as measurement errors become particularly large where sample size cannot even compensate for the degradation in accuracy. Notice that at levels 2 and 6 the function deviates from the above described phenomenon. As in the case of the expected total cost function of Figure 1, these anomalies are likely to be due to the discretization and grouping of information vectors.

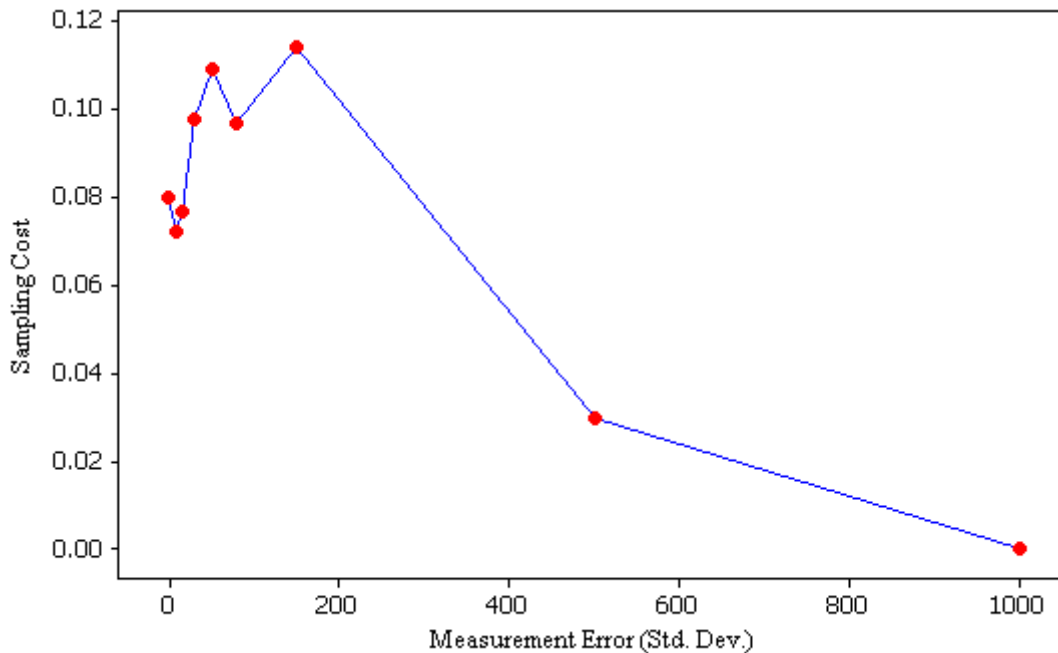


FIGURE 2 Expected inspection cost versus standard deviation of technology 1's measurement error.

### 3.3 Effect of annual budget constraint

The annual budget constraint is another important element of the formulation. Practically, the infrastructure agency budget is limited and often predetermined. In this application the condition state is not restricted to be maintained above a certain minimum level. However, the worse the condition state, the larger the user cost. As such the annual budget constraint plays a critical element in the optimization. Different values of annual budget constraints are specified at 100, 20, 15, 10, 8, 6, 4, 2, and 1 (in  $\$/m^2$ ).

The expected total cost at optimality along with each of its elements – user, IM&R, and terminal costs – in  $\$/m^2$  are plotted against the budget constraint applied on an annual basis (also in  $\$/m^2$ ) in Figure 3. As intuitively anticipated, a stricter annual budget constraint results in reduced expected IM&R cost and larger expected user and terminal cost. And, overall, an increase in the expected total cost is expected.

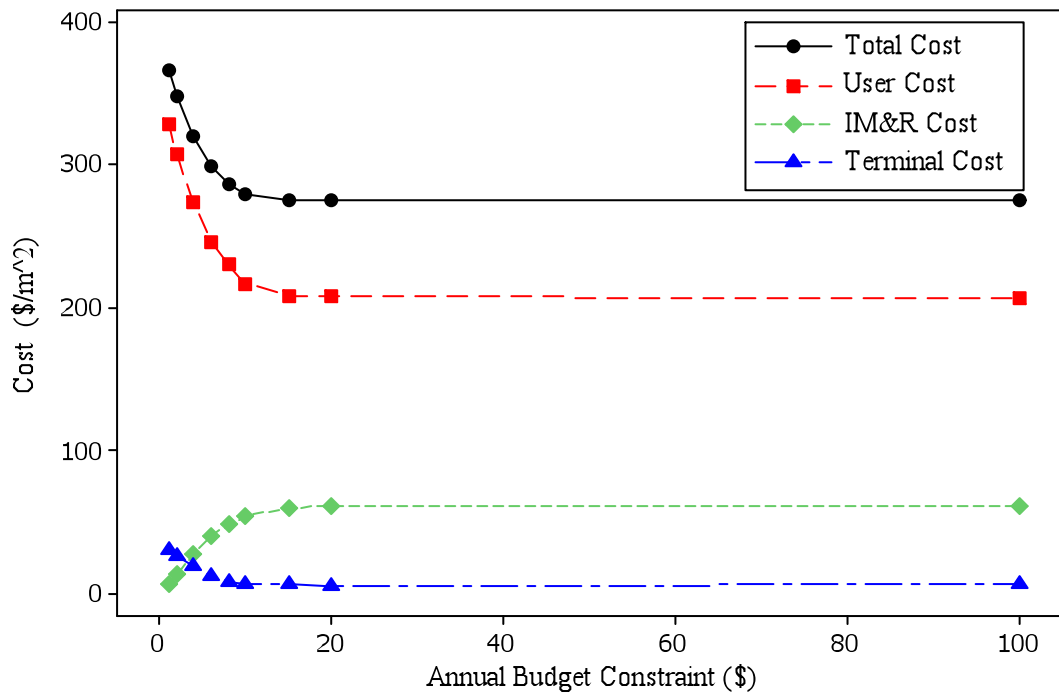


FIGURE 3 Expected total, user, IM&R, and terminal cost versus annual budget constraint.



### 3.4 Effect of optimal sampling

To determine the value of including sampling as decision variable with respect to the case where the sample size is pre-determined at a given level for all facilities repeatedly over time, the optimal solutions for the latter cases are determined and compared to that arrived at under the former case for the base scenario. Figure 4 shows the expected total cost at optimality under each of the above cases. The dashed horizontal line in Figure 4 indicates the expected total cost at optimality produced for the base scenario where the sample size is free to vary as a decision variable across all facilities and over time when determining the optimal IM&R policy. That is, the x-axis is not pertinent for this case; it is shown to facilitate the comparison.

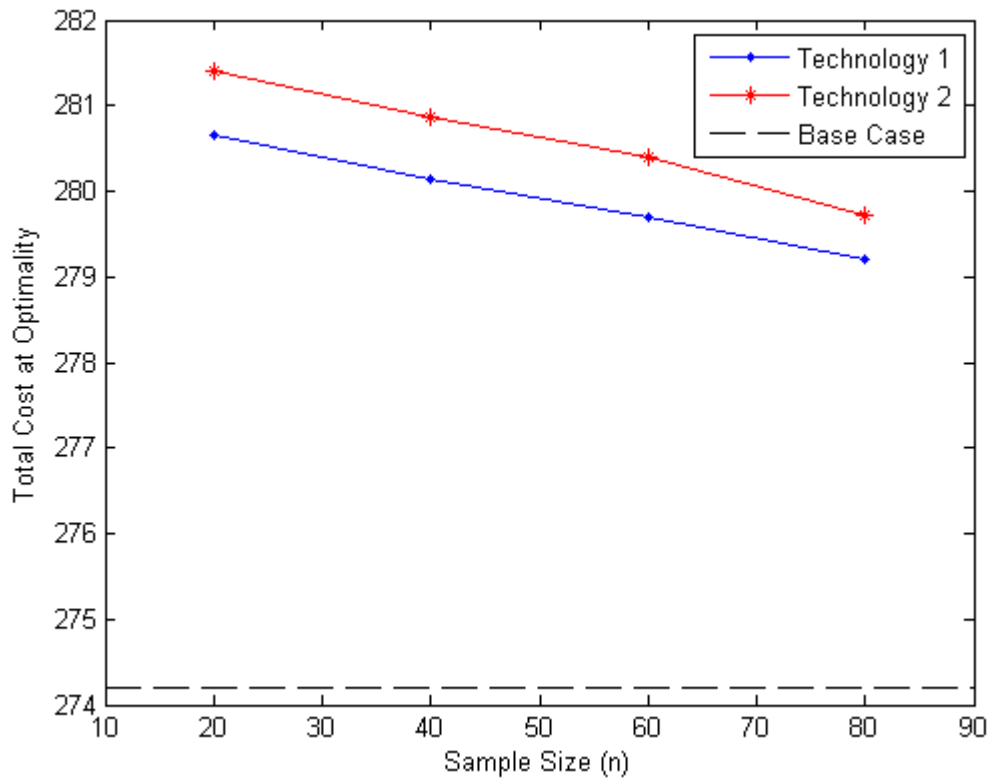


FIGURE 4 Expected total cost at optimality under pre-determined and optimal sample sizes.

The first set of points shown above the dashed line indicates the expected total costs at optimality when measurement technology 1 and the sample size value shown along the x-axis are set as inputs to determining the optimal M&R policy. The second set of points shown above the

previously described set correspond to the cases where measurement technology 2 is set as the technology of choice, and similarly, the sample size is set to the values shown along the x-axis. The relative difference between the expected total cost under the pre-determined sample size cases and the expected total costs when sample size and the choice of technology are optimized along with the M&R actions ranges between 1.7% to 2.2% suggesting an appreciable reduction in expected total cost when sampling is considered as a decision variable in light of the large costs typically associated with managing infrastructure networks.

Additional evaluations could be conducted considering the value of capturing the effect sampling uncertainty, either when the sample size is pre-determined or is determined optimally, with respect to the case where sampling and sampling uncertainty are ignored all together. However, such an evaluation requires a more involved simulation set-up, which is reserved for future research.

### 3.5 Sensitivity analysis

A sensitivity analysis is also conducted to explore the effect of various factors on the optimal solution. The factors and their corresponding values at different levels are shown in Table 3. The levels corresponding to the base scenario presented above are shown in italics. The other levels are based on additional scenarios developed to capture realistic ranges the various parameters could take.

In total, 1,458 scenarios are considered. The average expected total cost is calculated for each factor level, one factor at a time. The results are shown in Figure 5. User cost has the largest effect on the expected total cost. The annual budget constraint is the second most influential factor. The terminal cost and spatial correlation have similar effects on the expected total cost, in terms of magnitude. The effects of fixed inspection cost and unit inspection cost seem to be negligible.

Based on these results, not surprisingly, user cost is a key driver of IM&R decisions. Higher user costs result in more M&R actions to be taken to avoid increased used costs at poorer condition levels. Therefore, it is critical for agencies to assess and represent user costs accurately to avoid either over-spending on M&R actions (in the case where user costs are overestimated) or under-spending on M&R actions (in the case where use costs are underestimated) resulting in large user costs in actuality.

Again not surprisingly, the annual budget constraint is another key aspect of the problem. The lower the constraint, the more restricted the agency is in applying expensive M&R actions and the higher the user costs are due to the resulting poorer condition levels. In light of the quantification of the effect of the budget constraint on the expected total cost at optimality, it is worthwhile for agencies to determine such quantifications and present them to budget developers as an important input to setting budgets with a clear understanding of their implications on user costs.

TABLE 3 Factor Levels (L) and Values

Factor	L	Pavement Condition State			
		4	3	2	1
Correlation function	0	independent observation			
	1	$\rho(s) = \exp(-0.054271 \times s), \forall s > 0$			
	2	$\rho(s) = \exp(-0.026348 \times s), \forall s > 0$			
Fixed inspection cost (\$/m <sup>2</sup> )	0	0.0045 for 1 and 0.0042 for 2			
	1	0.0119 for 1 and 0.0093 for 2			
	2	0.0291 for 1 and 0.0152 for 2			
Unit cost ratio: tech 2 to tech 1	0	5.75			
	1	2.71			
	2	1.62			
User cost (\$/m <sup>2</sup> )	0	13.67	42	55.64	83.97
	1	18.47	56.74	75.16	113.4
	2	34.72	106.7	81.29	213.2
Terminal cost (\$/m <sup>2</sup> )	0	2.675	22.49	48.14	64.87
	1	0	0	0	0
Annual Budget Constraint	0	100			
	1	20			
	2	15			
	3	10			
	4	8			
	5	6			
	6	4			
	7	2			
	8	1			

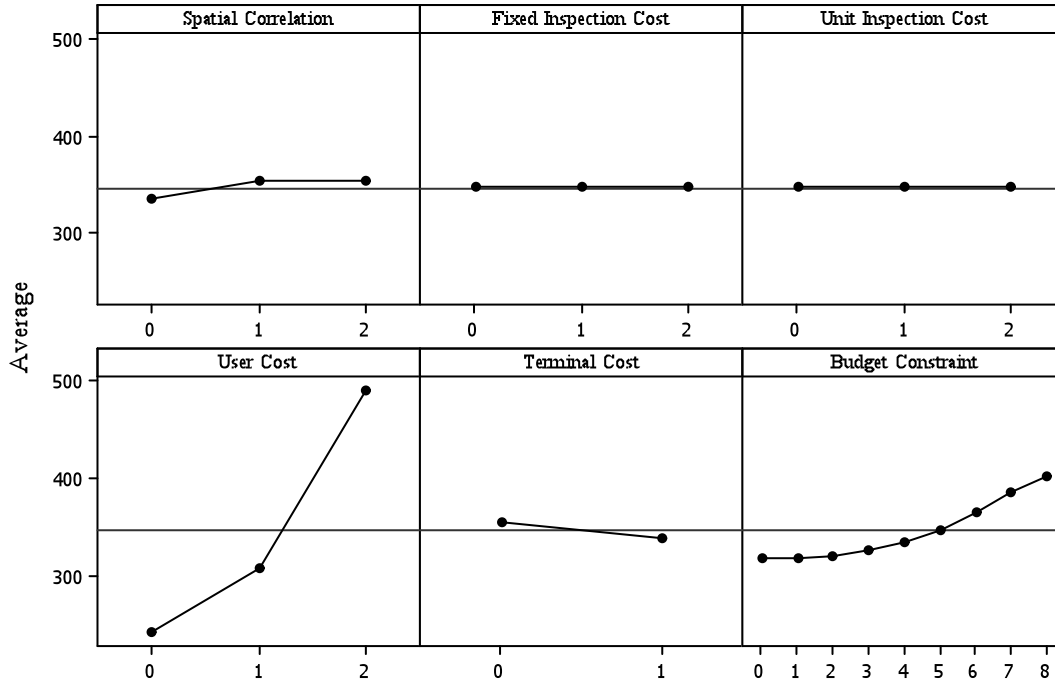


FIGURE 5 Average expected total cost versus factor levels.

It is valuable to note that the spatial correlation among adjacent observations along a facility, which has been shown to be present (Mishalani & Koutsopoulos 2002), does have an impact on the optimal solution and, thus, it is important for agencies to have a good understanding of the nature of this correlation.

As for terminal cost, the magnitude of the impact of ignoring such adjustments is not trivial and, therefore, it is crucial from a practical perspective for agencies not to overlook the equalization of service life concept, which has been theoretically established.

While the effect of inspection costs is negligible when it comes to the actual expected total cost at optimality, as is expected given the much larger magnitudes associated with M&R and user costs, it is important not to misinterpret this result as an indication of the lack of importance of considering inspection and sampling as part of the decision-making framework. Considering the uncertainty associated with inspection and sampling has important implications on the M&R actions that need to be taken. And, as has already been discussed, by setting the sample size at some predetermined levels, the expected total cost at optimality increases by appreciable amounts.

To investigate possible interaction between the above factors, interaction plots as shown in Figure 6 are analyzed by identifying functions for pairs of factors exhibiting increasing or diminishing differences for various combinations of levels. Several interactions are observed among pairs of user cost, budget constraint, terminal cost, and spatial correlation.

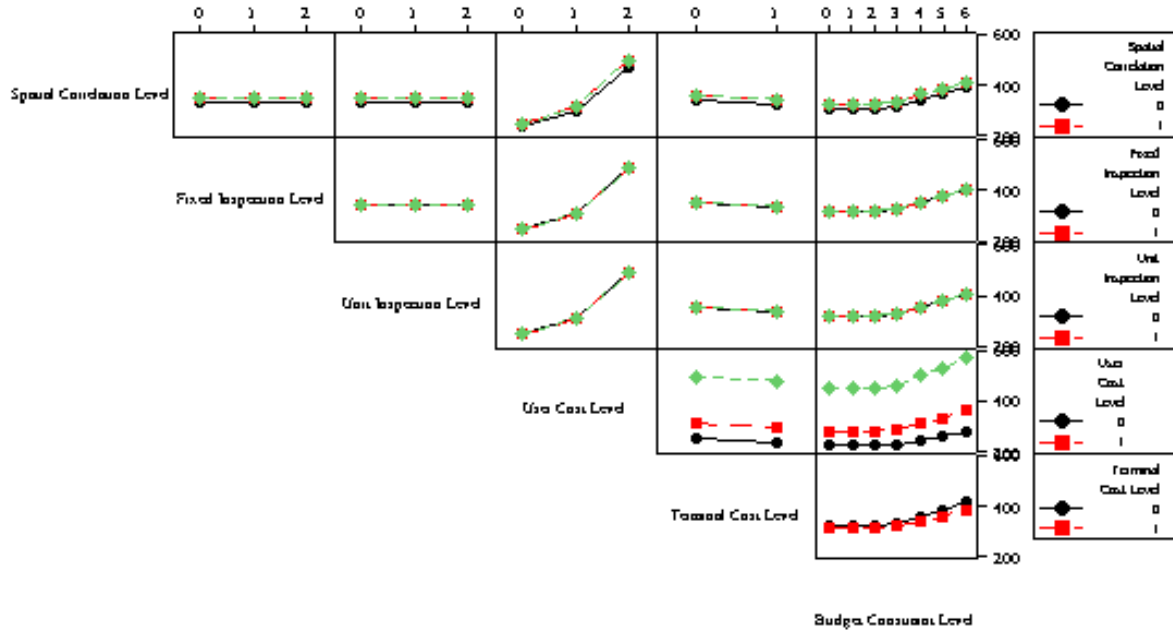


FIGURE 6 Pair-wise factor interaction plots.

Based on the above single factor discussion and the patterns exhibited in Figure 6, the most dominant and noteworthy interaction is that between the user cost and the budget constraint. The impact of the budget constraint on the optimal solution is larger under a higher user cost. That is, when the user cost parameters are high, the effect of the budget constraint on the optimal solution is magnified with respect to the case when the user cost parameters are relatively lower. This result is consistent with *a priori* expectations because a tight budget constraint limits the ability to apply M&R actions, resulting in user cost increasing non-linearly due to the non-linear nature of deterioration. Therefore, it is particularly critical for agencies to realistically assess and represent user cost and to avoid too restrictive budgets.

#### 4 CONCLUSION

This report presents a methodology developed to address the IM&R decision-making at the network level whereby the uncertainty due to condition sampling is captured and sample sizes over time and across the facilities forming the network are included as decision variables in the optimization. In addition, applications to hypothetical yet realistic set of scenarios are presented and discussed.

Based on examining the base scenario, it is clear that larger sample sizes can compensate for decreasing inspection accuracy up to a point where the degrading accuracy is so large, increasing the sample sizes does not offer much if any value. In addition, and not surprisingly, a stricter annual budget constraint will result in reduced expected IM&R cost and larger expected user and terminal costs. And, overall, an increase in the expected total cost is expected. Moreover, the effect of including sampling as a decision variable is found to be appreciable in terms of the expected total cost at optimality.

Based on examining all scenarios combined, the sensitivity analysis revealed that the user cost, annual budget constraint, terminal cost, and the spatial correlation function have appreciable impact on the optimal solution. Among these four factors, the impacts of the user cost and annual budget constraint are the most marked. Furthermore, these factors do interact with one another and the most notable interaction in terms of its magnitude and implications to agencies is that between user cost and the budget constraint.

The above results point to the importance of capturing sampling uncertainty and the sampling as a decision variable in addressing the IM&R problem at the network level in the presence of budget constraints, a set of considerations not addressed before. Moreover, the nature of the results, their interpretation, and their implications also offer contributions of note to both practitioners and researchers. Nevertheless, it is important to conduct a more extensive evaluation to quantify the value of capturing sampling uncertainty and including sampling as a decision variable along the lines of the effect of optimal sampling discussion in the previous section. Such a comprehensive evaluation has been conducted for the facility level problem in the absence of a budget constraint (Mishalani & Gong 2008), however, it remains to be undertaken as part of future research for the network level problem.

Clearly, the results are limited given the hypothetical, albeit realistic, nature of the scenarios considered in the numerical analyses presented in this report. Therefore, it is critical to demonstrate and assess the value of the developed methodology under field conditions to achieve a more comprehensive and realistic evaluation and possible refinements.

Another important limitation of the developed framework worth addressing is the absence of capturing facility interactions. Durango-Cohen & Sarutipand (2007) captured important interactions, however, they did not consider condition sampling. Developing a decision-making framework that simultaneously captures facility interactions and includes condition sample sizes across facilities and over time as decision variables would be worthwhile.

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