1. Assume that two potentials $V$ and $V'$ that differ by more than a constant give rise to the same electron density for a non-degenerate ground state. Now suppose that $\Phi$ is the eigenstate corresponding to $V$, and that $\Phi'$, corresponding to $V'$, must be less than in $\Phi$, and that for the rest, $T$ is the kinetic energy and $V$-electron repulsion. Now you see that the term with $V$ must cancel out on both sides because is the same for the two. Then the expectation value of $R$ in state $\Phi$, must be less than in $\Phi'$, and that for the rest, $T$ is the kinetic energy and $V$-electron repulsion. Now you see that the term with $V'$ cancels out on both sides.

2. ...Then the expectation value of $H$ prime in $\Phi$ prime,

must be less than in $\Phi$.

Write $H$ prime by adding two terms the first one with $V$ prime the next with the rest, $T$ is the kinetic energy and $V$-electron repulsion. Now you see that the term with $V'$ cancels out on both sides because is the same for the two. Then the expectation value of $R$ in state $\Phi$, must be less than in $\Phi'$, and that for the rest, $T$ is the kinetic energy and $V$-electron repulsion. Now you see that the term with $V'$ cancels out on both sides.

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