The Difficulty in Adding DC Power Flow Equations to Transmission Network Expansion

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In an AC power transmission network, the power flow along a line (i,j) can be approximated as

$$P(i,j) = \frac{1}{x0(i,j)}[\theta(i) - \theta(j)], \qquad (1)$$

$x0(i,j) =$ reactance between nodes i and j of existing network.
$\theta(i)$ = power flow angle of node i.
$P(i,j) =$ power flow along line (i,j).

In the case of transmission capacity expansion over time t(e.g., t is year index), the reactance along line (I,j) may be altered, resulting a highly non-linear term $\frac{1}{x0(i,j)+\Delta x(i,j)}$. This non-linearality remains true for the case of constructing a new transmission line.

More complication is that integers (e.g., binary variables) may be needed for signaling an expansion. For example, we may have to choose from a few commercial transmission lines with different voltage level and conductor sizes. In this case, Each binary variable may be assigned to one specific commercial transmission line, resulting in an expansion formula as this:

$$P(i,j) \le B1(i,j)P1\max + B2(i,j)P2\max + \ldots + Bn(i,j)Pn\max, \qquad (2)$$

$B1(i,j) =$ binary variable (0,1).
$P1$max=Type 1 conmmercial transmission cable.
Pnmax=Type n commercial tranmission cable.

And

$$P(i,j) = \frac{\theta(i) - \theta(j)}{x0(i,j) + x1(i,j)B1(i,j) + \ldots + xn(i,j)Bn(i,j)}, \qquad (3)$$

$xn(i,j) =$ reactance of commercial cable type n.

Notice that $xn(i,j)$ is not variable in (2) and (3) anymore. However, binary variables are at the denominator of (3), which also makes the equation highly non-linear.

The problem can be solved using the Benders decomposition or Lagrangean Relaxation. But the SAPP model is too big to solve with this DC power flow equations added.

Furthermore, there are a few direct current transmission lines in the SAPP network, which needs extra modeling techniques that are not explained in the above.

The next few pages show a transmission expansion problem with FACTs devices, published by Dr. Zuwei Yu in International Journal of Electric Power & Energy Systems in 2004. The complication is very clear.

The notation is summarized below:

h	hour index.
i, j	bus indices.
N	total number of busses.
s	seasonal (or month or week or day) index.
$H(i,s,h)$	representative hours for each h.
Hs	total number of hours used for season s.
Si	total number of seasons used for bus i.
$d(i,s,h)$	consumers' demand quantity at (i,s,h).
$P(i,s,h,d)$	price of the demand function (demand is a function of price).
$UF(i,s,h,d)$	utility function, the integration of $P(i,s,h,d)$.
$derate(i,s)$	generation capacity outage rate (forced and planned).
$q(i,s,h)$	power production quantity at (i,s,h).
$q_{\max}(i)$	upper limit of power production.
$q_{\min}(i)$	lower limit of power production.
$C(i,s,h)$	production cost of the generation plant at i.
$C0(i)$	constant of $C(i,s,h)$.
$C1(i)$	coefficient of the linear term of $C(i,s,h)$.
$C2(i)$	coefficient of the quadratic term of $C(i,s,h)$.
$Cex(i,j,s)$	cost of FACTS added at (i,j,s).
$C_C(i, j, s)$	cost of series capacitance at (i,j,s).
$C_C^0(i, j)$	constant of $C_C(i, j, s)$.
$C_C^1(i, j)$	coefficient of the linear term of $C_C(i, j, s)$.
$C_P(i, j, s)$	cost of phase shifter at (i,j,s).
$C_P^0(i, j, s)$	constant of $C_P(i, j, s)$.
$C_P^1(i, j, s)$	coefficient of the linear term of $C_p(i, j, s)$.
$F(i,j,s,h)$	real power flow in line (i,j).
$F_{\max}(i, j)$	original real power flow upper limit.
$F_{\max}(i, j)$	original real power flow lower limit.
$Fuc(i, j)$	line flow capacity upper limit allowed to expand due to capacitance addition.
$Fuw(i, j)$	line flow capacity upper limit allowed to expand due to phase shifter addition.
$Flc(i, j)$	line flow capacity lower limit allowed to expand due to capacitance addition.
$Flw(i, j)$	line flow capacity lower limit allowed to expand due to phase shifter addition.
$k_c(i, j)$	coefficient for line (i,j)'s flow limit expansion due to capacitance addition.
$\theta(i,j,s,h)$	power angle (in rads or degrees) between i and j without phase shifter.
$\theta_p(i, j, s, h,)$	power angle from the phase shifter.
$-x_c(i, j, s)$	reactance of the series capacitor(s) added to (i,j,s,h).
$-x_C^S(i, j, s, h)$	total reactance of the series capacitors added to (s,h).
$x(i,j)$	original reactance of the line (i,j).
$v(i,j,s)$	binary variable for series capacitor addition at (i,j,s).
$w(i,j,s)$	binary variable for phase shifter addition.

The impact of FACTs devices on reactive power is ignored to simplify the computation. The mathematical model is summarized below.

$$\begin{aligned} Max \sum_{s=1}^{Si} \Bigg\{ &\sum_{h=1}^{Hs} \sum_{i=1}^{N} H(i,s,h)[UF(i,s,h) - C(i,s,h)] \\ &- \sum_{i=1}^{N} \sum_{j=1}^{N} CRF \cdot Cex(i,j,s) \Bigg\} \end{aligned} \tag{1}$$

Subject to

$$UF(i,s,h,d) = \int_{0}^{d(i,s,h)} P(i,s,h,d')dd' \tag{1.1}$$

$$\begin{aligned} &C(i,s,h)=C0(i)derate(i,s)+C1(i)q(\mathrm{i},s,h) \\ &\quad +C2(i)\,q^2(i,s,h) \end{aligned} \tag{1.2}$$

$$Cex(i,j,s)= C_C(i,j,s) + C_P(i,j,s) \tag{1.3}$$

$$C_C(i,j,s) = C_C^0(i,j)\, v(i,j,s)+ C_C^1(i,j)\; x_c(i,j,s) \tag{1.4}$$

$$C_P(i,j,s) = C_P^0(i,j)\, w(i,j,s)+ C_P^1(i,j)\theta_p(i,j,s,h) \tag{1.5}$$

$$\begin{aligned} &F(i,j,s,h) = \\ &\quad \frac{[\theta(i,j,s,h) + \theta_P(i,j,s,h)][[x(i,j) - x_C^S(i,j,s,h)]}{[x(i,j) - x_C^S(i,j,s,h)]^2 + r(i,j)^2} \end{aligned} \tag{1.6}$$

$$\begin{aligned} &F(i,j,s,h) \le F\,\mathrm{max}(i,j) + \\ &\quad \sum_{t=1}^{s} [k_c(i,j)x_c(i,j,t)Fuc(i,j) + w(i,j,t)Fuw(i,j)] \end{aligned} \tag{1.7}$$

$$\begin{aligned} &F(i,j,s,h) \ge F\,\mathrm{min}(i,j) - \\ &\quad \sum_{t=1}^{s} [k_c(i,j)x_c(i,j,t)Flc(i,j) + w(i,j,t)Flw(i,j)] \end{aligned} \tag{1.8}$$

$$x_C^S(i,j,s,h) = \sum_{s'=1}^{s} x_C(i,j,s',h) \tag{1.9}$$

$$x_C^S(i,j,s,h) \le 0.8x(i,j) \tag{1.10}$$

$$\theta_P^{\mathrm{min}} \sum_{s'=1}^{s} w(i,j,s') \le \theta_P(i,j,s,h) \tag{1.11}$$

$$\theta_P(i,j,s,h) \le \theta_P^{\mathrm{max}} \sum_{s'=1}^{s} w(i,j,s') \tag{1.12}$$

$$q_{\mathrm{min}}(i)\, derate(i,s,h) \le q(\mathrm{i},s,h) \tag{1.13}$$

$$q(\mathrm{i},s,h) \le\; q_{\mathrm{max}}(i)\, derate(i,s,h) \tag{1.14}$$

$$d(i,s,h)=q(\mathrm{i},s,h)+ \sum_{j\in i}[F(i,j,s,h) + \frac{1}{2}r(i,j)F(i,j,s,h)^2] \tag{1.15}$$

$$\sum_{i=1}^{N} d(i,s,h) = \sum_{i=1}^{N} q(i,s,h) + \frac{1}{2}\sum_{i=}^{N}\sum_{j\in i} r(i,j)F(i,j,s,h)^2 \tag{1.16}$$

$$x_C(i,j,s) \le 10v(i,j,s) \tag{1.17}$$

In the following, we will explain the meanings of the major equations of this model. The objective function is to maximize the gross social welfare of consumers and producers minus the total costs including the costs of the FACTs devices.

Constraint (1.1) states that the utility function is the calculus integration of the inverse demand function (IDF), $P(i,s,h)$. For example, if $P(i,s,h)$ is linear, UF is a quadratic function. This IDF is characterized as a function of the decreasing nodal prince and can be approximated as a linear function or some other forms of functions [16].

The production cost function at bus i is quadratic in constraint (1.2). If the parameter $derate(i,s,h)$ is zero, the production quantity is zero and the cost is also zero. This is also true for the costs associated with the FACTS devices (see constraints (1.3), (1.4), (1.5), (1.11) and (1.12)). Note that the cost of a phase shifter can also be assumed not to depend on its power angles shifted [7].

Constraint (1.6) is the DC approximation of the real power flow in line (i,j) and the formulation can be found in [18]. The line reactance is modified by the addition of series capacitor(s). The power flower angle difference between buses i and j is also modified by the addition of a phase shifter. The effect of a phase shifter can be simplified to two net power injections at both end of the line [3]. The upper limit for adding series capacitance is suggested in [17] (see constraint (1.10)). In constraint (1.12) it can further be assumed that only one phase shifter is added throughout the planning horizon for a line. The flow limits of some lines can be increased to a certain degree and this is indicated by constraints (1.7) and (1.8). However, increased flow limit for any line over a long time may cause extra sag of the line eventually. Therefore, there must be a trade-off between increasing flow limit and limiting line sag because excessive sag of a line can cause safety problems and shorten the life of the line if the voltage level of the line is not increased.

Constraints (1.13) and (1.14) guarantee that power production at bus i is within its lower and upper limits. If there are multiple units at bus i, the model can be modified easily to resolve the problem. Constraint (1.15) equates demand at bus i with supply that includes power production at that bus, net power flows and losses. Losses are split into half-and-half and added to both ends of a line [18]. Finally, constraint (1.17) will force $x_c(i,j,s)$ to be zero if $v(i,j,s)$ is zero. Note that a scalar of 10 is added to the binary variable, $v(i,j,s)$, because per unit reactance potentially being added is usually less than one and 10 is a safe scalar to allow for exceptions.