For well-characterized experiments, data don't support the model conditional probability of observing the damping model cannot be rejected at 5% significance. What is $C_m$ data support the model; Validated $\pi$ Bayes factor. You Ling, Sankaran Mahadevan

Validation of Damping Prediction Surrogate Model

Motivation

• Are model predictions consistent with the experimental observations, given a defined level of uncertainty?

Surrogate model of damping factor

• Third order polynomial chaos expansion

$$C = \sum_{i=0}^{n} a_i \phi_i(t, g, f) + \epsilon_r$$

- $r$: thickness
- $a_i$: coefficient
- $\phi_i$: $i$th basis
- $f$: frequency
- $\epsilon_r$: residual

$$\phi(t, g, f) = \{1, t, g, f, t^2, g^2, f^2, t^* g, t f, g f^* f \}$$

• Estimated discretization error in damping factor is < 2%

Validation Data from Well-Characterized Experiments

Measurement made at center of beam for ring down data; beam excited electrostatically

• 7 Devices
• 4 pressures from 18784-66612 Pa
• 5 replicates of data
• Nickel beam
  - Length = 395.34 um
  - Width = 120 um

Graphical Comparison

Classical Hypothesis Testing

• Independent one-sample z-test

$$z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

- $z$: test statistic
- $\bar{x}$: sample mean of data
- $\mu_0$: mean of model prediction
- $s$: sample standard deviation of data
- $n$: number of data points

• For well-characterized experiments, $n = 1$ → What is $s$ value? Two options:
  - Assume $s$ = standard deviation of model prediction
  - Assume $s$ = standard deviation of measurement noise
• If $p$-value $= 2*\Phi(z) < 0.05$, the model is rejected at 5% significance level
• Damping model cannot be rejected at 5% significance level on most of the test points under higher pressures (62 out of 70)

Summary

• Validated the surrogate model of damping factor by comparing with experimental data, using graphical comparison, classical hypothesis testing, and Bayesian hypothesis testing

Model Validation Approaches

• Graphical comparison
  - Compare model predictions and experimental data graphically
• Classical hypothesis testing
  - z-test or t-test (to compare mean values)
  - Chi-Square test (to compare variances)
  - Chi-square, K-S, Anderson-Darling, Cramer tests (for distributions)
• Bayesian hypothesis testing
  - Bayes factor (applicable for well-characterized experiments and uncharacterized experiments)

Bayesian Hypothesis Testing

• Bayesian hypothesis testing

$$B = \frac{Pr(D | H_1, \text{model is correct}) \cdot \prod_i Pr(y_i | \theta_i)}{Pr(D | H_0, \text{model is incorrect}) \cdot \prod_i Pr(y_i | \theta_i)}$$

• Validation metric → Bayes factor
  - $B > 1$ → data support the model;
  - $B < 1$ → data don't support the model

• Bayes Factor for Well-Characterized Experiment

  - Construct conditional probability functions $\pi_s(y | \theta)$ and $Pr(C_{m0}|C_{m})$
  - Conditional PDF of $y$ under the null hypothesis $H_0$ given input $\theta$
  - $C_{m0} = N(G_t, g, f) + \epsilon_r$
  - $\epsilon_r \sim N(0, \sigma_r)$ → $C_{m} \sim N(G_t, g, f, \sigma_r) \rightarrow \pi_s(C_{m0}|G_t, g, f)$
  - Likelihood function → conditional probability of observing the data $y_i$ given model prediction $\theta$
  - $C_{m0} = N(0, \sigma_{obs}) \rightarrow C_{obs} \sim N(C_{m0}, \sigma_{obs}) \rightarrow Pr(C_{m0}|C_{m})$

• Calculations of Bayes factor

  - Four different surrogate models corresponding to four different pressures
  - $B$ is computed on every experimental site (140 in total, 35 for each pressure)
  - Models are supported by data on most of the experimental sites (121 out 140)

Figure 1: Example case of $B > 1$

Figure 2: Example case of $B < 1$