OBJECTIVE
The Boltzmann transport equation (BTE) can be used to describe the behavior of charge and/or energy carriers when the wavelength of the carrier is much smaller than the characteristic domain length. The BTE yields accurate results, but is computationally expensive to solve at very low Knudsen numbers (Kn=vg/L). However, in the limit of Kn -> 0, the BTE yields diffusive transport. We devise a hybrid BTE solver which solves a modified diffusive transport equation in the low Knudsen number limit, and a traditional BTE in the high Knudsen number limit. The methodology is implemented for a non-gray a phonon BTE and shown to perform well, but is extensible to BTEs for other carriers as well. Solution acceleration of 2-200 times is observed.

METHOD
We establish a cutoff Knudsen number , Knc, to separate discretized phonon bands into two groups: all phonons with Kn> Knc are solved with the BTE, all phonons with Kn< Knc are solved with a modified Fourier equation. All Fourier equations and the lattice temperature equation are solved simultaneously using a block algebraic multigrid method. With a new lattice temperature, the BTEs are then solved sequentially. The process is iterated until convergence.

BTE
\[
\nabla \cdot \left( \tilde{v}_{a,p} e^{a}_{n,p} \right) = \frac{e^{a}_{n,p} - e^{a}_{n,p}}{\tau_{a,p}}
\]

Modified Fourier Equation (MFE)
\[
- k_{a,p} \nabla T_{a,p} = \frac{C_v}{\tau_{a,p}} (T_a - T_L) \quad k_{a,p} = C_v \frac{\delta v_{a,p}}{3} \quad h = \frac{\delta v_{a,p} C_v}{3}
\]

Block Structure of MFEs and Lattice Temperature Equations
\[
A_P T_L = \sum_{ab} A_{ab} T_{ab} + B_P \quad \text{(Band 1)}
\]
\[
A_{P2} T_{L2} = \sum_{ab} A_{a2,b} T_{a2,b} + B_{P2} + A_{P2} T_{L2} \quad \text{(Band 2)}
\]
\[
T_{L} = \sum_{s} R_{s} T_{Ls} + \sum_{s} R_{s} T_{Ps} \quad \text{(Lattice Eq)}
\]

VERIFICATION
Single band comparison to exact solution

Solution Acceleration

<table>
<thead>
<tr>
<th>Method</th>
<th>All-BTE</th>
<th>Hybrid All-BTE/Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>26662.62</td>
<td>1612.86</td>
</tr>
<tr>
<td>Iterations</td>
<td>126</td>
<td>14</td>
</tr>
</tbody>
</table>

Mesh dependence tests were performed separately for the full band all-BTE and hybrid solvers. A Richardson’s extrapolate was used to determine that a 1% error in dimensionless heat flux was obtained using a 120x120 mesh for the all-BTE solver. For the hybrid solver the dimensionless heat flux was found using mesh sizes of 40x40, 80x80, and 160x160. A 1% change in dimensionless heat flux occurred from 80x80 to 160x160.

Conclusions:
• Diffusion approximation is sufficient for appropriate Kn.
• Reduced dimensionality allows for a more implicit solution method.
• Block structured algebraic multigrid speeds up linear solve.
• Little accuracy is sacrificed for a drastic reduction in computational effort.
• Same methodology may be applied to electron and gas molecule BTE.