A J2 Polycrystalline Constitutive Model
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Abstract:
We simplify a standard single-crystal plasticity model to formulate a J2-theory. Size effects are still taken into account. The resultant model is solved numerically for uniaxial stress. Numerical results are compared with real uniaxial stress experiments performed on LIGA nickel components.

Single-crystal plasticity model with size effects:
In a single crystal the total plastic strain is assumed to evolve according to
\[
\dot{\varepsilon}^p = \sum_{i=1}^{N} \dot{\lambda}_i |\tau_i| \mathbf{m}_i = \sum_{i=1}^{N} \dot{\lambda}_i \text{sgn}(\tau_i) \mathbf{m}_i
\]

where \( \dot{\lambda}_i \geq 0 \) are the plastic slips. \( \mathbf{m}_i \) are the Schmidt tensors and the \( \tau_i \) are the resolved shear stresses per slip system \( i \). The yielding of the crystal is described by Schmid’s law

\[
f_i(\sigma, \lambda) = |\tau_i| - \tau^c_i(\lambda) = 0, \quad i = 1, 2, \ldots, N
\]

where \( \tau^c_i(\lambda) \) is the critical resolved shear stress on the \( i \)th slip system.

\[
h_{ij} = \frac{a \mu n_i}{\rho_i} \left( \frac{\tau^c_i(\lambda)}{\tau^0_i(\lambda)} \right)^{1/2} \left( \cosh \left( \frac{\tau^0_i(\lambda)}{\tau^c_i(\lambda)} \right) - 1 \right) \delta_{ij}.
\]

\[
n_i = \sum_{j=1}^{N} a_{ij} \rho_j
\]

\[
\rho_i = \rho_{sat} \frac{\lambda_i}{h} \sqrt{\frac{(1 - 2x_2/h)^2}{(x_2/h)(1 - x_2/h)}} + \rho_0
\]

Experiments:
Hemker & Last, 2001 (LIGA Ni)

Model Results/MPM Results (Verification):

Simplified J2 model with size effects:
In a J2 model, the plastic strain is assumed to evolve according to

\[
\dot{\varepsilon}^p = \lambda \frac{\sigma^\text{eff}}{(\sigma^\text{eff} ; \sigma^\text{eff})^{1/2}}
\]

where \( \sigma^\text{eff} \) is the effective stress. Assume the critical resolved shear stress is identical in all slip systems. The resolved shear stress is replaced by the effective stress and the yield criterion takes the form

\[
f(\sigma, \lambda) = \frac{3}{2} \sigma^\text{eff} \sigma^\text{eff}^{1/2} - \tau^c(\lambda)
\]

where \( \tau^c(0) = \sigma^y \) is the initial yield stress that includes the Hall-Petch size effect. (The initial yield depends on the manufacturing process, the geometry and the grain size.) The critical stress evolves according to:

\[
\tau^c(\lambda) = \frac{1}{2} A \mu \frac{\tau^0}{\tau^c(\lambda)} \lambda
\]

with

\[
\tau^0 = a \sqrt{A \mu b \rho} \quad \rho = 2 \rho_{sat} \lambda b / h + \rho_0
\]

References: