Objective: To Study Fluid-Structure-Electrostatics Interaction in MEMS Through the Use of the Finite Volume Method

- Use cell-based finite volume method to characterize fluid-structure-electrostatics interactions within a single comprehensive numerical framework
- Use second-order spatial discretization and first order temporal discretization of the integral conservation equation governing elastic solid mechanics
- Apply the structural solver to study deformations of MEMS structures under electrostatic actuation
- Electrostatic actuation treated as a surface force and added as a source term in the governing equation for structural deformation
- Fluid-structure-electrostatics coupling implemented in a sequential manner
- Arbitrary Lagrangian Eulerian technique for handling deforming meshes.

Mesh Motion Strategy

- Calculate shortest distance \( d(i,j) \) of every fluid node \((i,j)\) to moving walls
- Identify the wall node \((i_{\text{wall}},j_{\text{wall}})\) closest to the fluid node \((i,j)\)
- Closest wall nodes found by KD-Tree based search algorithm
- Calculate shortest distance

\[
d^2 = \sum_{k=1}^{nnodes} \left( \frac{1}{d(i,k)} - \frac{1}{d_{\text{max}}} \right)
\]

- Rigid grid near the wall and far away from the wall
- Rest of the areas are elastic and easily deformed
- Subsequently, grid smoothed using spring-mass analogy

\[
\delta t = \frac{1}{\sum k_i} \sum_{i=1}^{n_{\text{nodes}}} \sum_{k=1}^{n_{\text{nodes}}} \left( \frac{1}{d(i,k)} \right) \cdot \left( \frac{1}{ds_{ij}} \right) \cdot \delta s_{ij}
\]

- \( k_i = 1/|ds_{ij}| \)
- \( ds_{ij} \) = distance between nodes \( i \) and \( j \)

Structured-Electrostatics Interaction in MEMS

- Top Electode details

| Beam Length | 400 \( \mu \)m |
| Beam Thickness | 4 \( \mu \)m |
| Width (w) | \( 1\text{mm}(2D) \) |
| Young’s modulus | 200 GPa |
| Poisson’s ratio | 0.31 |

\[ \nabla \cdot \phi = 0 \quad \text{in air} \]

\[ \sigma_t = -\varepsilon A \frac{\partial \phi}{\partial \eta} \]

\[ \begin{align*}
F_{\text{elec}} &= \int_{0}^{L} \left[ \varepsilon_0 \nabla \cdot (\varepsilon \nabla \phi) \right] \, dx \\
&= \text{electrostatic force on the top electrode}
\end{align*} \]

- Top electrode assumed an equipotential surface
- Electrostatic force treated as a surface force on the top electrode and is assumed to be a non-linear function of the gap width

Quasi-static Pull-In Voltage

- Quasi-static pull-in voltage of 181V matches well with analytical solution
- Significantly larger pull-in voltage for partial bottom electrode

Fluid-Structure-Electrostatics Interaction

- Apply electrostatic force and fluid stress as boundary conditions on solid
- Solve structural model and calculate beam deformation
- Deform fluid mesh and calculate convection flux through the CV faces
- Solve electrostatic model and re-compute electrostatic force
- Apply mesh velocity as boundary condition at the fluid-beam interface
- Solve fluid model and go to next time step

Structural Deformation Due To Electrostatic Actuation

- Top electrode treated as a fixed-fixed beam

Analytical Solution (Euler-Bernoulli Theory)

\[ W_j = \frac{\omega^2 (L-x)^2}{24EI} \]

\( \omega \) = force per unit length

Future Work

- Implement second order temporal discretization of the structural solver
- Implement contact physics, creep and plasticity in the physical models for structural deformation
- Explore the possibility of a single coupled fluid-structure-electrostatics solver as an alternative to sequential coupling