Sources of uncertainties in MEMS
- Material properties - Young’s modulus, etc
- Geometrical features - dimensions, gaps, etc
- Operating environment, boundary conditions

Discontinuities and nonlinearities
- Electrostatic MEMS exhibit pull-in instability
- Nonlinear dependence on input parameters
- Need for adaptively refining uncertainty analysis to capture these phenomena

Objectives
- Quantify effect of uncertainties on performance parameters
- Adaptively refine solution in regions of large error using domain decomposition
- Find sensitive input parameters
- Employ uncertainty quantification data in the design of MEMS

Introduction

Problem Formulation

Coupled Electro-Mechanical problem

Mechanical analysis
- Large deformation elasticity

Electrostatic analysis
- Boundary integral formulation

Stochastic Formulation

Stochastic formulation uses random variables and fields to model uncertain parameters:

\[ u(x), \sigma(x) \rightarrow u(x, \xi), \sigma(x, \xi) \]

\( \xi = [\xi_1, \ldots, \xi_n] \) iid random variables

Stochastic collocation method
- Approximation: Approximate unknown stochastic solution using hierarchical interpolation on a sparse Smolyak grid

Example: MEMS Switch

Effect of uncertain Young’s modulus and gap

Sampling: Solve deterministic problem at nodes of sparse grid
Post-processing: Compute statistics

Adaptive sparse grid collocation
- Domain decomposition: Adaptively split random domain into subdomains, \( \Gamma_i \)
- Hierarchical surpluses: Use to estimate
- Local contribution to global error
- More sensitive random dimensions

Define splitting criterion
Split subdomain \( s \), IF \( J_s \geq J_1 \) \( J_s = \text{Pr}(\xi \in \Gamma_s) \)
Split along dimension \( i \), IF \( \gamma_i \geq \gamma_1 \) \( \gamma_i = \max_{1 \leq k \leq n} (u_i^k), i = 1, \ldots, n \)

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