Bayesian Calibration of Dielectric Charging Model Parameters

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Calibration Procedure

- 6 dimensional parameter set θ
  - μ*, φ_B, γ, σ, ε_{inf}, N_T
- Data
  - 12 Input Combinations of T and V
  - 4 Replicates Time Series at Each Setting
  - ~190 Time Points in Each Time Series (0-100 s)
- Measurement Noise – Use replicates at each time point to directly calculate standard deviation, σ_f, for each input ε_f = N(0, σ_f)
- Model Discrepancy – Take mean value of replicates at each time point for each input combination ε_{inf} = Y_m - Y_D
- Since ε_{inf} is a function of input, it must be well described as ε_{inf} = f(T, V)
- Measurement Noise = N(μ_{inf}, σ_{inf})
- Likelihood – Gaussian distribution of difference between model prediction and observation
  - L(θ) \equiv Pr(Y_m(t) - Y_D(t) | ε_{inf}, ε_D)
- Prior Distribution – Uniform distribution (uninformed prior) over a known physical range of the parameters
  - π(θ) = C \forall θ ∈ Θ

Bayesian Calibration

- Bayes Theorem
  - \pi(\theta | D) = \frac{L(\theta) \pi(\theta)}{\int L(\theta) \pi(\theta) d\theta}
  - \theta: model parameter
  - D: experimental data
  - L(\theta): likelihood function of \theta
  - \pi(\theta): prior PDF of \theta
  - \pi(\theta | D): posterior PDF of \theta

Problem Features

- Calibration of Time Series Data – includes multiple time series w.r.t. replication and variation of input settings
- Inclusion of Model Form Error – both mean and variance are functions of the inputs
- Simultaneous Parameter Calibration – underlying model parameters are calibrated along with parameters of error mean and variance models

Experimental Data

- SNA2 - R1C3M1, V=10V
- T=300K, 320K, 340K, 360K
- Current (A) vs. Time (s)

- SNA2 - R1C3M1, V=15V
- T=300K, 320K, 340K, 360K
- Current (A) vs. Time (s)

Calibration Results

- Experiment Data
  - Least Squares Estimate
  - Constant Variance Mean Value
  - Constant Variance Standard Dev
  - Non-Constant Variance Mean Value
  - Non-Constant Variance Standard Dev

Model Prediction

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