Motivation and Goals

- Measure Transient Dynamics of RF MEMS Switch.
- Investigate new techniques to estimate important system parameters of MEMS

Transient Measurements

- Initial transient measurements have commenced.
- Measuring Pull-in, Pull-out, and switch bounce vs. various voltage inputs

Parameter Estimation

- Decouple Young’s modulus from residual stress, initial curvature, and boundary conditions using linearized beam equation with stretching term:

\[
\rho \frac{d^2W(x,t)}{dt^2} + E W_{xxyy} + N W_{yy} = E A \left( \frac{d^2W(x,t)}{dx^2} \right)^2 + \frac{E A}{2} \frac{d^2W(x,t)}{dx^2} \frac{d^2W(x,t)}{dx^2} + N W_{yy} = 0
\]

Linearize:

\[
\phi''(x) = -\alpha^2 \phi''(x) - \beta^2 \phi(x) = \frac{1}{2} \phi(x)\frac{d^2\phi(x)}{dx^2} \frac{d^2\phi(x)}{dx^2}
\]

Let: \( W_0 = \frac{1}{2} (1 - \cos(2\pi x)) \)

Show:

\( \alpha = \alpha_0 + \delta \alpha \)

Natural Frequency vs. \( \alpha_0 \), fixed-fixed BC: \( \gamma_{n,m}^2 \)

Natural Frequency vs. \( \alpha_0 \), fixed-free BC: \( \gamma_{n,m}^2 \)

Beam Cross Section

Estimate Torsional Flexibility from Measured Modes:

\[
T_{L} = \frac{\phi_0(0)}{\phi_0(0)}
\]

Solve EVP:

\[
\frac{\phi''(x)}{\phi''(x) - 1} = \frac{1}{2} \phi(x)\frac{d^2\phi(x)}{dx^2} \frac{d^2\phi(x)}{dx^2}
\]

Calculate thickness from Torsion mode 1

- Roark’s Formulas for Stress and Strain 7th ed.

And Continuous Vibrations- S.S. Rao

\[
\rho I_p \frac{d^2\theta(x,t)}{dt^2} = C \frac{d^2\theta(x,t)}{dx^2} \rightarrow \text{EOM with warping correction-}
\]

Assume FIXED FIXED BC, C is torsional rigidity, \( \rho \) is mass density, \( I_p \) is polar moment of inertia

\[
\omega_n = \sqrt{\frac{C}{\rho I_p L}} \quad \text{or} \quad C = GJ
\]

\[
G = \frac{C}{k(2a)^{3/2}} \quad \text{or} \quad J = \frac{C}{G}
\]

\[
k = \frac{1}{3} \left[ \frac{192a}{\pi^3} \sum_{i=1,3,5,\ldots} \frac{1}{i^3} \tanh \left( \frac{a}{b} \right) \right]
\]

\[
c = \frac{\omega_n^2 \rho I_p}{k(2a)^{3/2}} E = 2G(1 + \nu)
\]

Parameter Estimation

- Calculate thickness from Torsion mode 1

Die 9 10 ms sweep

Pullin-Pullout