Motivation and Objectives
Unsteady micro-scale flows within moving solids of complex geometries with large-scale deformation and high dynamic rate displacement require unsteady fluid-structure interaction modeling.

Lagrangian-Eulerian methods for fluid-structure interaction are based on conformal meshes and require incessant, time-consuming re-meshing. Fixed-grid methods provide an alternative to re-meshing and the Immerged Boundary Method (IBM) has recently found increasing use for microflows as well as turbulent flows [Ref. 1, 2].

Main Objectives: Develop immersed boundary method formulation for Boltzmann ESBGK equations and apply to transient, large displacement, gas damping simulations of MEMS switch with opening and closing gaps.

BGK Equation
\[
\begin{align*}
\frac{df}{dt} + c_r \frac{df}{dx} + c_t \frac{df}{dt} &= \frac{f-f_0}{\tau} = \frac{1}{\tau} \frac{P}{\mu} \\
\end{align*}
\]

IBM-BGK: Method 1 (Least Squares Interpolation)

1) Find \( f_{\text{outgoing}} \) using 2nd order Least Squares Interpolation [Ref. 3]

2) Conservation of mass flux at solid face
\[
\sum_{\text{IB}} c I_{\text{IB}} + \sum_{\text{s}} c I_{\text{s}} \exp \left( \frac{(v-y)\Delta t}{T} \right) = 0
\]

Step 3: Interpolate \( f_{\text{coming}} \) at IB face
Step 4: Solve A\( \delta f + R = 0 \) for all directions (\( c_x, c_y, c_z \))

1D BGK Code Verification

1D Verification Results: FVM-BGK vs IBM-BGK

Extension to 2D
Relaxation-type gas kinetic BGK flux schemes are widely used in 3d subsonic and supersonic flows. [Ref. 4] For a 2D formulation steps 2.4 include a weighted sum of distribution function from cells/IB-faces in the sampling radius
\[
\begin{align*}
\tilde{f}_{\text{IB}} &= \sum_i \frac{w_i}{1/(v-x_i)} \left( f_{\text{IB}} + \frac{f_{\text{IB}} - f_{\text{IB}}}{T} \exp \left( \frac{-\Delta x}{T} \right) \right)
\end{align*}
\]

Conclusions
- Error with respect to a full BGK simulation for both methods decrease with decreasing \( \Delta x_{\text{IB}} \)
- Error is minimum when \( K_{\text{IB}} > K_{\text{IB}} \Rightarrow \Delta x_{\text{IB}} < \Delta x \)
- Method 1: Error \( \uparrow \) if \( \Delta x < \Delta x_{\text{IB}} \), min of \( \approx 1.6\% \) when \( K_{\text{IB}} = 3.8 \)
- Method 2: Error is less than 1\% when \( K_{\text{IB}} = 1.9 \)
- Method of relaxation looks promising and it can easily be extended for both ESBGK and 2D/3D problems.

References