**Design of Electrostatically Actuated MEMS Under Uncertainties**

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**Introduction**

**Sources of uncertainties in MEMS**
- Material properties - Young’s modulus etc.
- Geometrical features - dimensions, gaps etc.
- Operating environment
- Boundary conditions

**Objectives**
- To design reliable and efficient MEMS devices,
- quantify the effect of various uncertain parameters on performance parameters,
- employ the uncertainty quantification data to identify critical design parameters.

**Stochastic framework for MEMS:**
Based on stochastic collocation approach,
- straightforward to implement, and,
- can be orders of magnitude faster than traditional approaches, such as Monte-Carlo (MC) method.

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**Problem formulation**

**Coupled Electro-Mechanical problem**

**Mechanical analysis**
Large deformation elasticity problem

**Electrostatic analysis**
Boundary integral formulation

**Deterministic coupled problem**

\[ \mathcal{L}(u, \sigma; x) = 0 \quad x \in \Omega \]

Solved using Finite Element Method (FEM) and Boundary Element Method (BEM)

**Stochastic formulation**

Stochastic formulation uses random variables and fields to model uncertain parameters:

- Deterministic: \( u(x), \sigma(x) \)
- Stochastic: \( u(x, \xi), \sigma(x, \xi) \)

\( \xi = [\xi_1, \ldots, \xi_n] \) n-id random variables

**Stochastic coupled problem**

\[ \mathcal{L}(u, \sigma; x, \xi) = 0 \quad x \in \Omega \times \Xi \]

**Stochastic collocation method**
- Basic idea is to approximate the unknown stochastic solution using interpolation

\[ \{u(x, \xi), \sigma(x, \xi)\} = \sum_{j=1}^{K} \{u(x, \xi^j), \sigma(x, \xi^j)\} \psi_j(\xi) \]

**Sampling**
- Solve deterministic problem at sparse grid nodes obtained using Smolyak algorithm

**Design under uncertainties**

**Example: MEMS switch**

Effect of uncertain Young’s modulus and gap
- Assumed to be uniformly distributed.

\( E_0 = 169 \text{ GPa} \)
\( \Delta E = 0.1E_0 \)
\( g_0 = 1 \mu m \)
\( \Delta g = 0.1g_0 \)

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**Figures**

- MEMS lateral comb drive
- MEMS beam under deformation
- MEMS lateral comb drive
- Schematic of a MEMS switch
- Worst case and mean pull-in behavior
- Sensitivity of vertical tip displacement w.r.t. \( E \) and \( g \)
- Probability of pull-in