Achieving Full-Scale Simulation: Part I – Coarse-Grained Model

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Operation of Radio Frequency MEMS

Off state: Low capacitance, On state: high capacitance

Low loss, excellent quality factor ...
CV characteristics of a MEMS device

- Source of instability
- Point of instability
- Geometry of instability
- Energy dissipation

![Graph showing CV characteristics](image)
Reduced Order Model

\[ m \frac{d^2 y}{dt^2} = F_{elec(down)} - k(y_0 - y) - b \frac{dy}{dt} \]

Net force:
\[ F = \frac{\varepsilon_0 \varepsilon_r^2 AV^2}{2(y_d + \varepsilon_r y)^2} \]

MATLAB simulation
Pull-in and Pullout Voltage

\[ \frac{dF_E}{dy} = 2A/y^3 = k \]

\[ y_s = 2/3y_0 \]

\[ V_{pi} = \sqrt{\frac{8k\varepsilon A}{27\gamma}} \]

\[ E = \frac{1}{2}CV_A^2 \]

\[ F_E = \frac{dE}{dy} = \frac{1}{2}V_A^2 \frac{dC}{dy} \]

\[ C = \frac{\varepsilon_0 A}{y} \]

\[ F_s = k(y_0 - y) \]

\[ F_{s}, F_{E} \]

\[ E_{diss} = \frac{1}{2} \frac{\varepsilon V_{pi}^2 A}{y_f} - \frac{ky_f^2}{2} \]
1D analysis is good, but not good enough …

![Graph showing capacitance vs. voltage with two sets of curves: Initial and Final. The graph on the right shows the experimental data with a peak at around -10 V and another peak at around 10 V.]
Mechanics of cantilevers - fixed beam mechanics

\[ m \frac{d^2 y}{dt^2} = F_{elec} - k(y_0 - y) - b \frac{dy}{dt} \]

\[ \rho_l \frac{\partial^2 y}{\partial t^2} = F_{elec}(x, y) - \frac{EI}{1 - \nu^2} \frac{\partial^4 y}{\partial x^4} - b \frac{\partial y}{\partial t} \]
Outline

Basics of MEMs Operation

1D vs. 2D MEMs Ideal MEMS

Nonideal MEMs:

- Viscoelastic Creep
- Squeeze Film Damping
- Dielectric Charging

Sensitivity Analysis

Conclusions
(2) Nonideal effects: Damping

\[ \rho \frac{\partial^2 y}{\partial t^2} = -\frac{EI}{1 - \nu^2} \frac{\partial^4 y}{\partial x^4} - b \frac{\partial y}{\partial t} + F_{\text{elec}} \]

\[ b = \frac{F}{L_m \frac{\Delta y}{\Delta t}} = \left( \frac{W_m}{y} \right)^3 \eta \]

\[ \lambda = \frac{k_B T}{\sqrt{2\pi d^2 P}} \]

\[ b = \frac{A \left( \frac{W_m}{y} \right)^c H_m}{1 + B \left( \frac{W_m}{y} \right)^d \left( \frac{\lambda}{W_m} \right)^e} \]
Dynamics of MEMS: 2D result

\[
\rho l \frac{\partial^2 y}{\partial t^2} = -\frac{EI}{1 - \nu^2} \frac{\partial^4 y}{\partial x^4} - b \frac{\partial y}{\partial t} + F_{elec}
\]

- Damping is not constant throughout the motion
- Damping is pressure dependent

Time for pulling down the membrane increases with pressure
(3) Nonideal Effect: Dielectric Charging

Charge accumulation in traps. Note the polarity.
Shift of CV curve, stiction, and lifetime

Switch fails to open

Switch fails to open

Capacitance (nF)

Voltage (V)

0
-50 -25 0 25 50

In the start

Voltage (V)

0 5 10 15

1e-3 1 1e3

Pull-in voltage

Pull-out voltage

PRISM
NSA Center for Prediction of Reliability, Integrity and Survivability of Microsystems
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- Basics of MEMs Operation
- 1D vs. 2D MEMs Ideal MEMS
- Nonideal MEMs: Creep, Damping, Charging
- Verification and Validation
- Sensitivity Analysis
- Conclusions
Many Equations and Many Parameters

Tunnel current from contacts to trap

\[ q \Delta y \frac{dn_t(x)}{dt} = \frac{4\pi \alpha \Delta y N}{h^3} \left( 1 - \frac{n_t(y)}{N} \right) E_{high} \int T(E, y) \beta(E, y) S(E) f(E) dE \]

\[ S(E) = k_B T \ln \left( 1 + \exp \left( -\frac{E - E_F}{k_B T} \right) \right) \]

\[ T(E, y) = \exp \left( -\frac{2}{\hbar} \int_0^y \sqrt{2m^* m_e (\Phi_b - q \Phi(y) - E)} dy \right) \]

\[ f(E) = \frac{1}{1 + \exp \left( \frac{E - E_F}{k_B T} \right)} \]

\[ \beta(E, y) = \exp \left( -\frac{\Phi_b - q \Phi(y) - \Phi_i - E}{k_B T} \right) \]

Equation of membrane dynamics

\[ \rho \frac{\partial^2 y}{\partial t^2} = F_{elec}(x, y) - \frac{EI}{1 - v^2} \frac{\partial^4 y}{\partial x^4} - b \frac{\partial y}{\partial t} \]

Poisson Equation

\[ V_0 \varepsilon_d \varepsilon_0 - \int_0^y \int_0^x \rho(y) dy dx \]

\[ E_a = \frac{\varepsilon_d \varepsilon_0 x_a + \varepsilon_a \varepsilon_0 x_d}{\varepsilon_d \varepsilon_0} \]

\[ V_d(y) = \frac{\varepsilon_a \varepsilon_0 E_a + \int_0^y \rho(y) dy}{\varepsilon_d \varepsilon_0} \]

\[ V(x) = V(x - \Delta x) + V_d(y - \Delta y) \Delta y \]

Squeeze film damping coefficient

\[ b = A \left( \frac{W_m}{y} \right)^c H_m \]

\[ 1 + B \left( \frac{W_m}{y} \right)^d \left( \frac{\lambda}{W_m} \right)^e \]
**Verification: Dynamic Model**

\[ \rho_l \frac{\partial^2 y}{\partial t^2} + \frac{EI}{1 - \nu^2} \frac{\partial^4 y}{\partial x^4} = 0 \]

\[ y(x,t) = \sum_{n=1}^{\text{Number of Modes}} c_n \phi_n(x) \cos(\omega_n t) \]

\[ \phi_n(x) = (\cosh(\gamma_n x) - \cos(\gamma_n x)) - k_n (\sinh(\gamma_n x) - \sin(\gamma_n x)) \]

\[ \omega_n^2 = \left( \frac{\gamma_n}{L} \right)^4 \frac{EI}{(1 - \nu^2) \rho_l} \]
Verification: Dielectric Charging

\[ V_{PI}(0) = \sqrt{\frac{483 EI}{(1-v^2)L_m^3}} \times \frac{8 (y_d + \varepsilon_r y_0)^3}{27 W\varepsilon_r^3 \varepsilon_0} \]

\[ V_{PO}(0) = \sqrt{\frac{333 EI}{(1-v^2)L_m^3}} \times \frac{2 y_0^{1.881} y_d^{1.07}}{W\varepsilon_r^{1.17} \varepsilon_0} \]

\[ n_T(x) = \frac{A_{IN}(x) N_T}{A_{SUM}(x)} \left( 1 - \exp \left( -\frac{t}{q} A_{SUM}(x) \right) \right) \]

\[ V_{PO}(t) = V_{PO}(0) - \frac{q \int_0^{y_d} x n_T(t, x) dx}{\varepsilon_r \varepsilon_0} \]

FS: Full numerical self-consistent simulation with Euler-Bernoulli solution

CG: Coarse grained model
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**Input Parameters**

Sensitivity

\[
\frac{v_{\text{max}} - v_{\text{min}}}{v_{\text{avg}}} \times \frac{u_{\text{avg}}}{u_{\text{max}} - u_{\text{min}}}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>St. Deviation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_m): Membrane Length ((\mu)m)</td>
<td>500</td>
<td>10</td>
<td>Measurements</td>
</tr>
<tr>
<td>(H_m): Membrane Thickness ((\mu)m)</td>
<td>1.85</td>
<td>0.3</td>
<td>Measurements</td>
</tr>
<tr>
<td>(E): Young modulus (GPa)</td>
<td>350</td>
<td>80</td>
<td>Measurements</td>
</tr>
<tr>
<td>(y_0): Airgap ((\mu)m)</td>
<td>3.5</td>
<td>0.26</td>
<td>Measurements</td>
</tr>
<tr>
<td>(y_d): Dielectric Thickness (nm)</td>
<td>191</td>
<td>16</td>
<td>Measurements</td>
</tr>
<tr>
<td>(\varepsilon_r): Dielectric constant (-)</td>
<td>7.9</td>
<td>0.5</td>
<td>Literature/Guess</td>
</tr>
<tr>
<td>(\phi_B): Barrier Height (eV)</td>
<td>1.5</td>
<td>0.2</td>
<td>Measurements</td>
</tr>
<tr>
<td>(N_T): Trap density (cm(^{-3}))</td>
<td>2e18</td>
<td>1e18</td>
<td>Measurements</td>
</tr>
<tr>
<td>(\sigma): Capture cross section (cm(^2))</td>
<td>1e-17</td>
<td>5e-18</td>
<td>Literature/Guess</td>
</tr>
<tr>
<td>(m^*): Effective mass (-)</td>
<td>0.5</td>
<td>0.2</td>
<td>Measurements</td>
</tr>
<tr>
<td>(\gamma): FP attempt frequency (s(^{-1}))</td>
<td>1e12</td>
<td>5e11</td>
<td>Literature</td>
</tr>
</tbody>
</table>
Sensitivity Analysis

Performance

Reliability

Dynamics

V_{PI} \quad V_{PO} \quad T_{FAIL} \quad T_{FAIL} \quad V_{PI}
UQ for Membrane Dynamics (Dakota)

Global PDF, Operating Voltage=80V

Membrane properties
And air gap (Lm, Hm, y0, and E)

Combination of membrane
& dielectric properties (Hm, y_d, and $\epsilon_r$)
Operation Frequency = 1KHz

**UQ for Dielectric Charging (Dakota)**

**Pull in voltage**

- Mean ($\mu$): 48.5V
- Standard Deviation ($\sigma$): 10.7V

**Pull out voltage**

- Mean ($\mu$): 8.9V
- Standard Deviation ($\sigma$): 1.9V

**Failure time**

- Number of cycles ($10^x$)

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**Membrane properties & air gap (Lm, Hm, y0, and E)**

**all parameters**

**Dielectric properties ($\gamma_d$, $\phi_B$, m, and $N_T$)**

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**Operation Frequency = 1KHz**
Conclusions

- The basic properties of MEMs are easily understood in 1D model.

- 2D model is necessary to understand the specific shape of C-V characteristics and details of dielectric charge injection.

- Many higher order effects like creep, damping, and dielectric charging have significant effect on operation of practical MEMS switches.

- Among numerous model parameters, performance metrics like pull-in voltage and pull-in time are most sensitive to air-gap, membrane thickness, and Young’s modulus.

- Reliability however is dictated essentially by dielectric properties related to barrier height, trap depth, and film thickness have important effect on lifetime prediction.

- The coarse-grain model provides a solid foundation for MEMs operation.