What are the Limits of the Discrete Instrument Model?

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The Big Picture

- It would be great if we could use mathematical models to tell us
  - Why good instruments are good
  - How a design change will affect the response of an instrument
  - How a newly designed instrument will sound

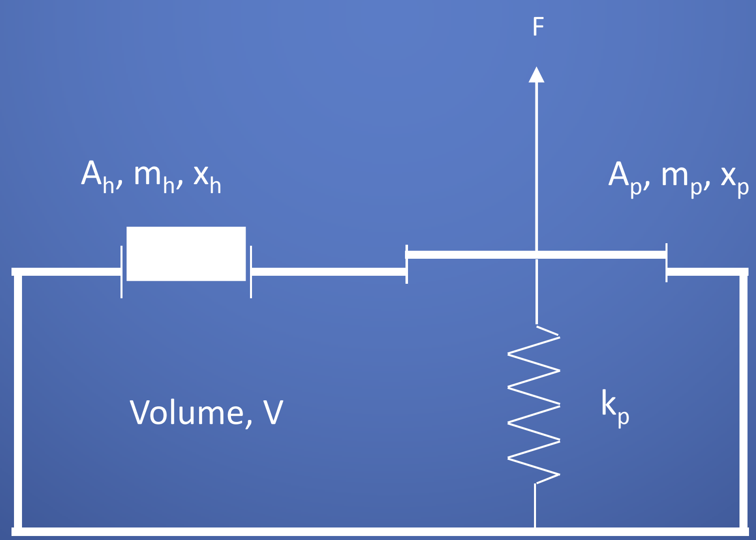
Unfortunately, the underlying physics are very complex and models that attempt to capture it are cumbersome to work with (e.g., big FE models)

Simple models with a few degrees of freedom can be very helpful in understanding low frequency behavior

Discrete Instrument Model

- Model was proposed by Christensen and Vistisen in 1980
- Makes intuitive sense
- A very good teaching tool

A_h	Area of sound hole
m_h	Mass of the air column moving through sound hole
x_h	Position of air column moving through sound hole
V	Volume of enclosed air
k_p	Stiffness of top plate
F	Force applied to top plate
A_p	Effective area of top plate
m_p	Effective mass of top plate
x_p	Position of top plate

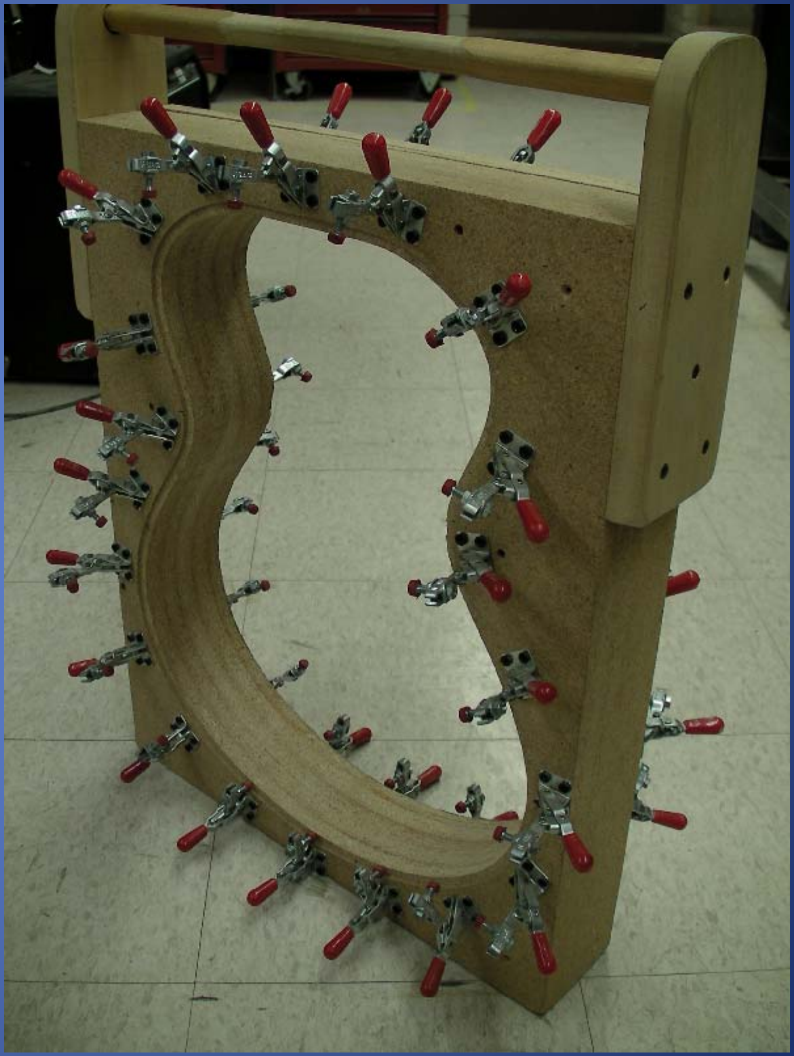


What Are the Limits?

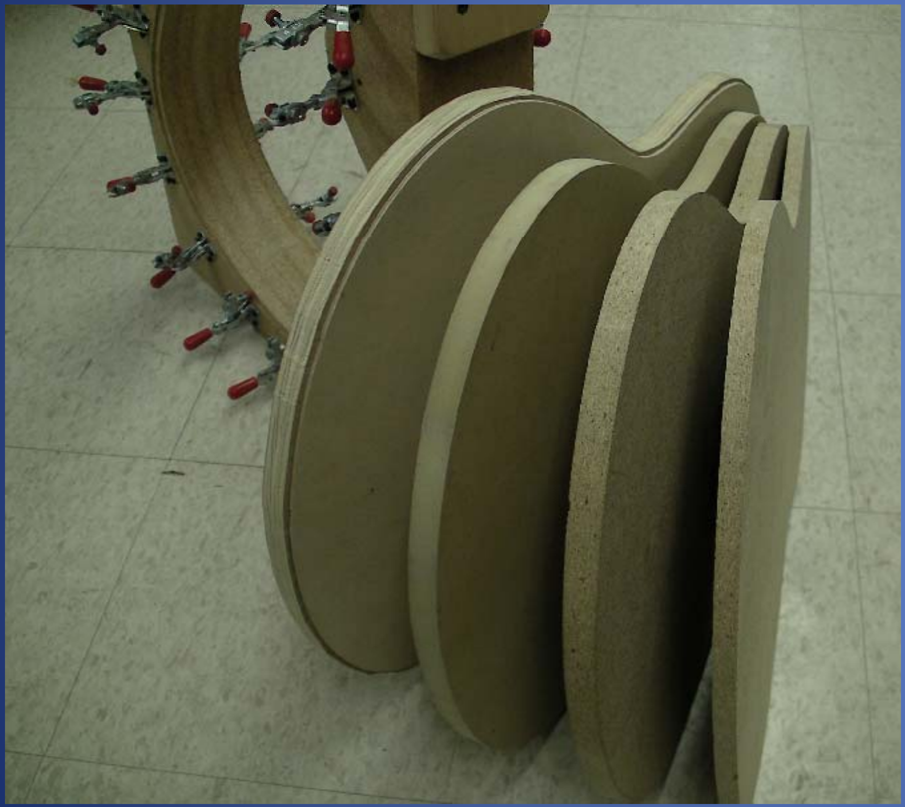
- Discrete model is a good tool for understanding acoustic-structural interaction
- Is used qualitatively in instrument design
- How well does it work quantitatively?
- Do we need something different?

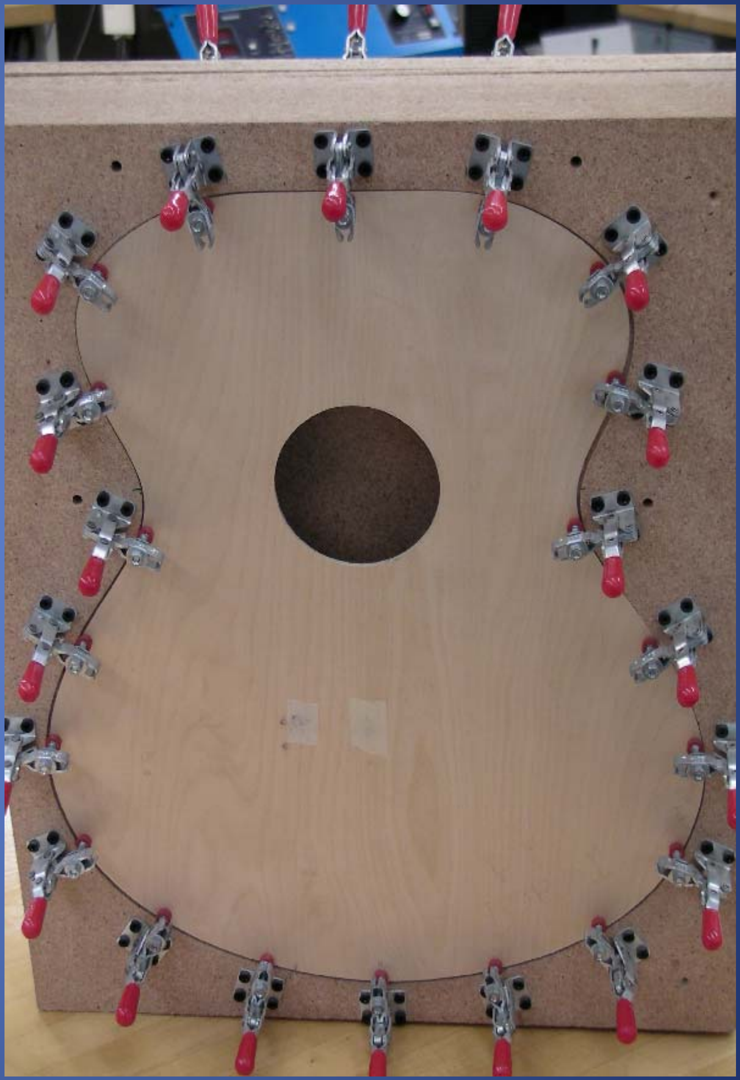
Test Fixture

- Designed to implement the assumptions in the 2-DOF Discrete Model
- Top and back plates are easy to change
- Spacers can be added to change interior volume



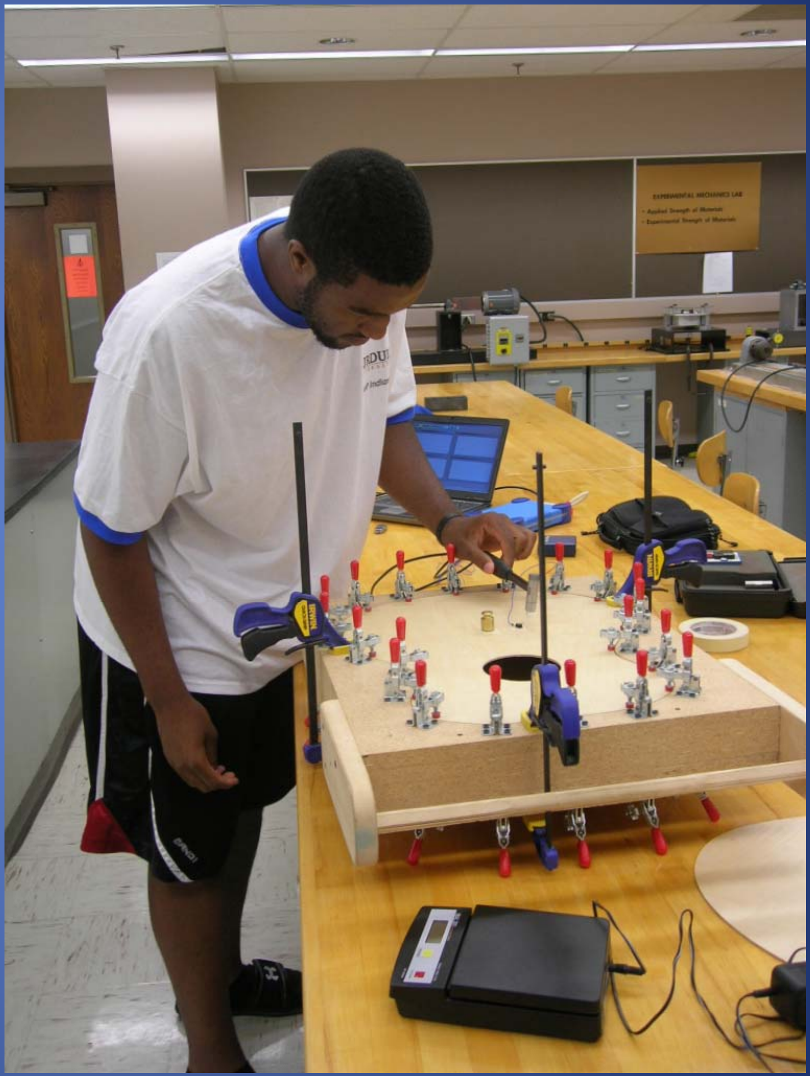
Top Plate and Spacers





Testing

- Driving point FRFs captured using impact hammer and miniature accelerometer
- Five taps averaged for each FRF
- Coherence near 100% for most measurements
- Several test conditions evaluated

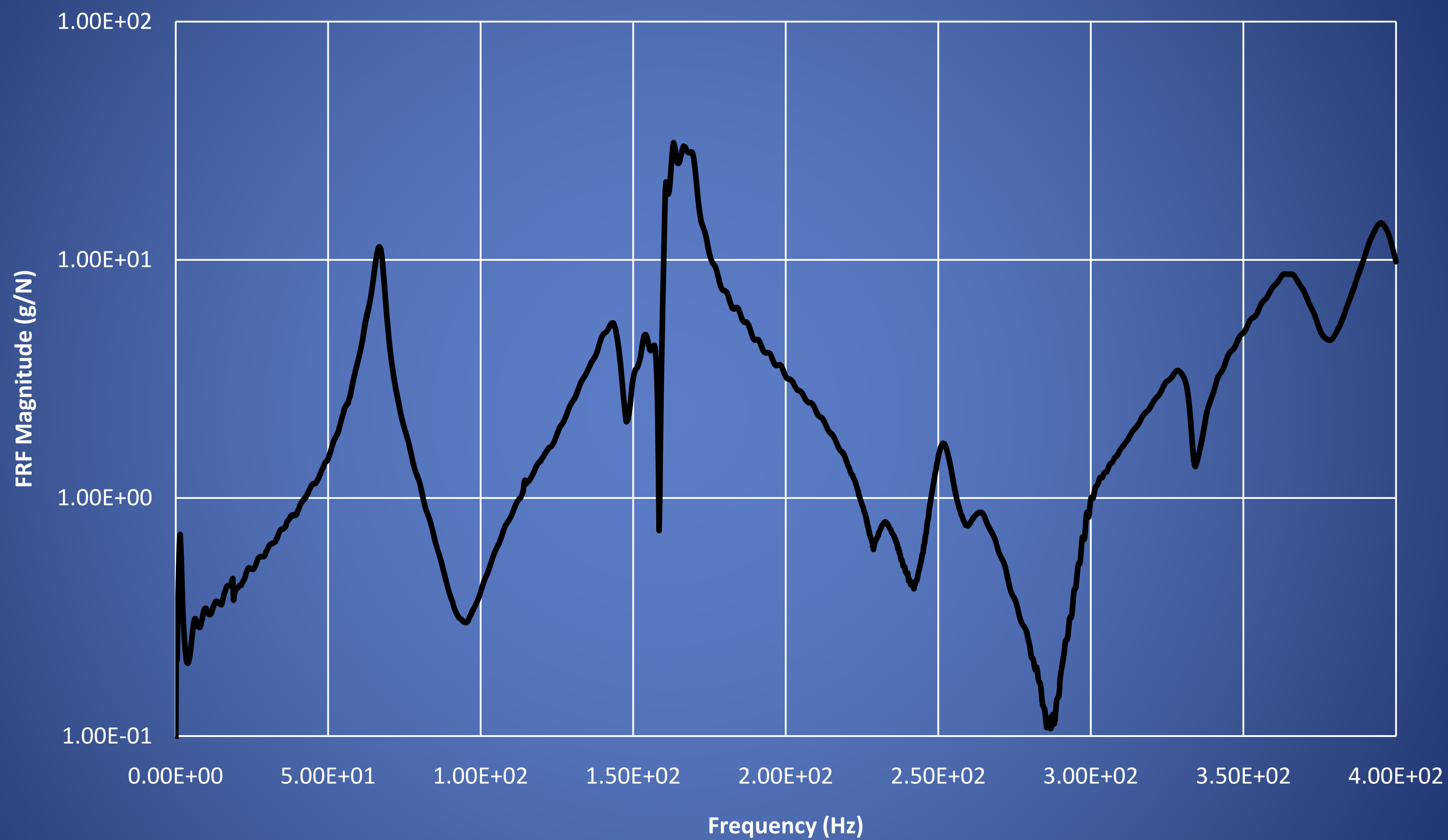


Summary of Test Conditions

Number of Spacers	Body Volume (cm^3)	No Additional Mass	20g Additional Mass	50g Additional Mass
0	15450	x	X	x
0.67	13240	x		
1	12140	x	X	x
1.67	9933	x		
2	8829	x	X	x
2.67	6622	x		
3	5518	x	X	x

Additional masses provide data for tuning discrete model

Typical Frequency Response Function

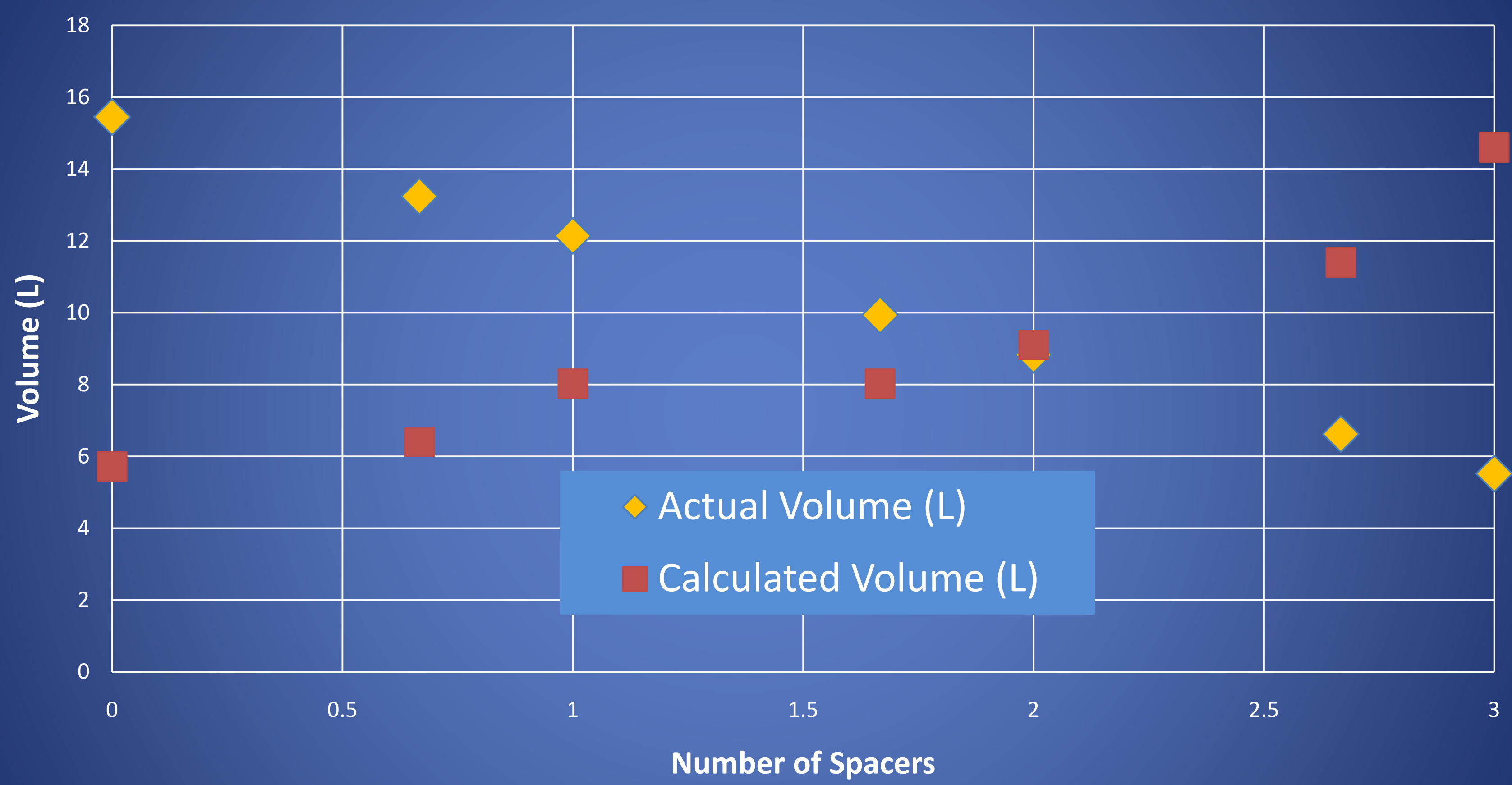


Data Used for Tuning Model

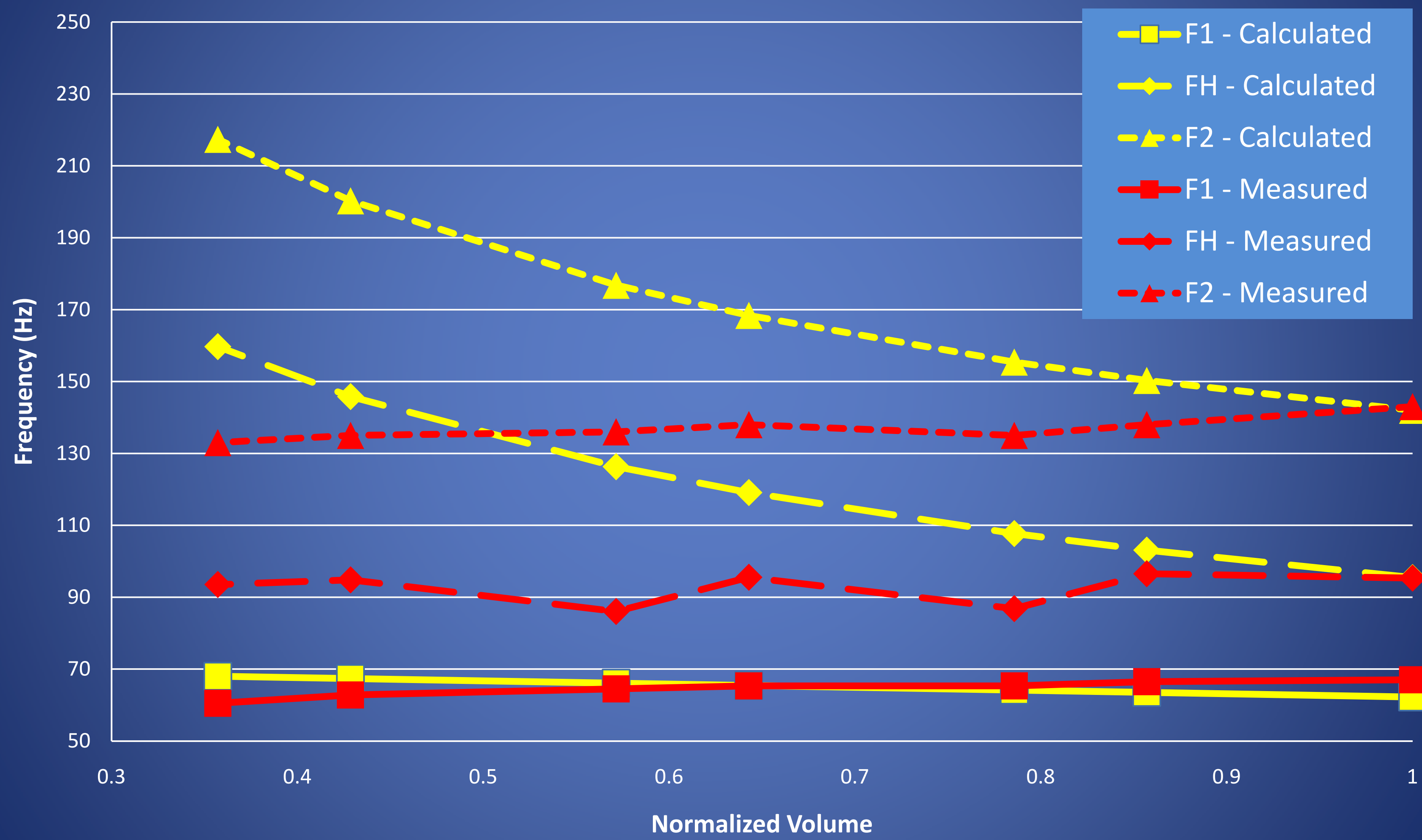
Additional Mass (gm)	Type	F_1 (Hz)	Ratio	F_H (Hz)	Ratio	F_2 (Hz)	Ratio
0	Measured	67.0		95.3		143	
	Calculated	62.21	1.08	95.47	0.998	142.0	1.01
20	Measured	59.3		90.8		136	
	Calculated	61.44	0.965	95.47	0.951	138.6	0.981
50	Measured	53.8		85.8		128	
	Calculated	60.32	0.892	95.47	0.899	134.1	0.955

Red boxes indicate data not used for model tuning
Additional mass does not affect F_H in discrete model

Effective Volume Required to Match F_1



Summary of Measured and Computed Frequencies



Analytical Check

- There are closed form expressions for derivatives of eigenvalues
- A simple condition must be met for resonant frequency to decrease with increasing volume
- Condition not met for tuned model, so $\partial\lambda/\partial V > 0$

$$\frac{\partial \lambda_n}{\partial V} = -\frac{c^2 \rho}{V^2} \frac{\left(A_p^2 x_{1n}^2 + 2 A_h A_p x_{1n} x_{2n} + A_h^2 x_{2n}^2\right)}{m_p x_{1n}^2 + m_h x_{2n}^2}$$

Eigenvalue Derivative

$$x_{1n} x_{2n} < -\frac{1}{2 A_h A_p} \left(A_p^2 x_{1n}^2 + A_h^2 x_{2n}^2\right)$$

Condition required for $\partial\lambda/\partial V < 0$

Conclusions

- Measurements do not match analytical predictions
- Measurements do not exhibit expected trends
- Is it possible that a real instrument acts more like the model than a structure designed expressly to match the model?

We wish to thank the session organizers and ASA for the invitation to attend this conference and LSAMP for providing the funding that made the trip possible

A Parting Thought

Tim Shaw, Lead Engineer at Fender demonstrates the ‘Laying on of Hands’ – the means by which mojo (a mystical and unquantifiable goodness) is imparted to an instrument