CHAPTER 3
CAPACITY CONSTRAINTS

The capacity constraints insure that each plant's generation does not exceed the properly derated installed capacity for the day types in the model.

This chapter will be organized by type of facility, considering the capacity constraints in sequence of (a) old thermal sites, (b) old hydro sites, (c) new thermal sites, (d) new hydro sites, (e) old transmission lines, (f) new transmission lines, and (g) new and old pumped storage.

In all cases, the model will allow no additions to capacity in 2000, the initial year of the horizon; those the SAPP delegates indicated were under way at the July 1998 workshop will be treated as capacity already installed. Thus, only the optimal dispatch problem is solved in 2000, capacities being fixed: this means the year 2000 can be run in isolation from the rest of the program, since no decision in 2000 affects subsequent years. These projected initial capacities as of year 2000 are included in the following tables:

- Thermal: Table *PGOinit(z,i)* (Appendix IV, Section 13).
- Hydro: Table *HOinit(z,ih)* (Appendix V, Section 4).
- Transmission: Table *PFOinit(z,zp)* (Appendix III, Section 1).
- Pumped Storage: Table *PGPSOinit(z)* (Appendix V, Section 8).

These generating capacities are based on installed capacity by plant name given in Tables 1.1 and 1.2; initial transmission capacities are given in Figure 1.2.

When calculating the available capacity in a time period, the normal capacity must be derated by the expected forced [*FOR(z,i)*] and unforced [*UFOR(z,i)*] outage rates. Since forced outages by their nature, can occur anytime, peak or off-peak, the nominal capacity must be derated by the expected forced outage rate for all operating periods, peak or off-peak. On the other hand, the timing of unforced, or planned outages for preventive maintenance can be chosen by the utility. Since generating capacity is more valuable during periods of peak demand than during off-peak periods, unforced outages are scheduled during off-peak periods. However, unforced outage rates are given in terms

of annual percentage. Hence, the off-peak unforced outage rate used in the model is slightly larger than the annual rate, since the peak unforced outage rate is zero, and the weighted average of the two rates equals the annual rate.

This approach requires that the thermal capacity constraints enter the model by day type, since peak capacity will only be derated by the forced outage rate while off-peak capacity should be derated by both forced and unforced outage rates. Hydro and transmission capacity will be derated only by forced outage rates, as will be explained below.

Since the model is a long-term model, the likelihood that plants will deteriorate due to old age, and be unable to generate their initial capacities must be taken into account.

The model does this by introducing a user specified decay parameter scalar *Decay*, a function of the technology type (e.g., *DecayHN*, *DecayHO*, etc., all found in Section 13 of Appendix VI), giving the yearly percent reduction in generating capability, which applies to each site in each country, which is introduced into the capacity constraint.

Note there should be a connection between the user specified values of *UFOR*, *FOR*, and *Decay*; the higher *UFOR* is, the lower the other two should be to recognize that the purpose of preventive maintenance is to reduce forced outages and equipment decay.

A generic capacity constraint requires that generation in period *ty*, *PG(ty,ts,td,th,z,i)*, respect the available installed capacity in period *ty*, equal to the sum of the initial capacity (adjusted for decay) plus the capacity added up to and including *ty*; again, (adjusted for decay), properly derated for peak and off-peak periods:

$$PG(ty,ts,td,th,z,i) \le \Big[(Initial\ \ capacity)(decay\ factor)+ \sum_{\tau=1}^{ty}(\ capacity\ added\ in\ \tau)(decay\ factor\ for\ capacity\ added\ in\ \tau)\Big] derate\ for\ day\ type$$

Assuming decay takes place at a constant percent per period *ty*, the right hand side of the expression would be:

$$\left[(Initial\ Capacity)(1\text{-}Decay)^{ty} + \sum_{\tau=1}^{ty} (Capacity\ added\ in\ \tau)(1\text{-}Decay)^{ty\text{-}\tau} \right] derate\ \ for\ day\ type$$

Since the decay rate is a yearly rate, the decay exponent must by the period *ty* times the year/period, *n*.

For example: if *ty* = *3* and *n*=2 (two years per period), the right hand side of the expression would be:

$$\left[(Initial\ Capacity)(1\text{-}Decay)^{3} + (Capacity\ added\ in\ 1)(1\text{-}Decay)^{2} \right.$$
$$\left. +(Capacity\ added\ in\ 2)(1\text{-}Decay)^{1} + (Capacity\ added\ in\ 3)(1\text{-}Decay)^{0} \right] derate$$

The model allows users to remove control of the selection of specific projects from the optimization program, and specify that a project must be:

(a) <u>started</u> in a given period (one unit completed),

(b) <u>started</u> and <u>finished</u> in a given period (entire project completed),

(c) <u>started</u> before or at a given period (one unit completed),

(d) <u>started</u> and <u>finished</u> before or at a given period (entire project completed),

(e) <u>prevented</u> from being started until after a given period.

This option is available for all new transmission and generation projects, as well as expansion of existing hydro and pumped hydro units: it is not available for expansion of existing thermal units.

The model allows for the decommissioning of all existing generating units – thermal, hydro, and pumped storage.

Finally, the model allows for the user to set minimum and maximum load factors (expressed as a decimal fraction of existing capacity) for all existing and new generating units and transmission lines; only existing and new pumped storage does not have this control option.

All this is summarized in Table 3.1.

Table 3.1 User Operating Options

	Before/At/After Constraints	Min/Max Load Constraints	Decommission Constraints
Existing thermal	no	yes	yes
New thermal	yes	yes	no
Existing hydro	yes	yes	yes
New hydro	yes	yes	no
Existing pumped storage	no	no	yes
New pumped storage	yes	no	no
Existing lines	no	yes	no
New lines	yes	yes	no

3.1 Old Thermal Sites

During all times within a given year *ty*, the generation levels given by the variables *PG(ty,ts,td,th,z,i)* must respect the properly derated site capacity available in that year. Initial MW capacities for the sites are given in Table *PGOinit(z,i)* in Appendix IV, Section 13. The value of the decision variable *PGOexp(tyb,z,i)*, which indicates how many units of the MW size given in Table *PGOexpstep(z,i)* (Appendix IV, Section 4) are to be constructed in year *tyb*, determines the amount of new MW capacity installed at site *i* in year *ty*.

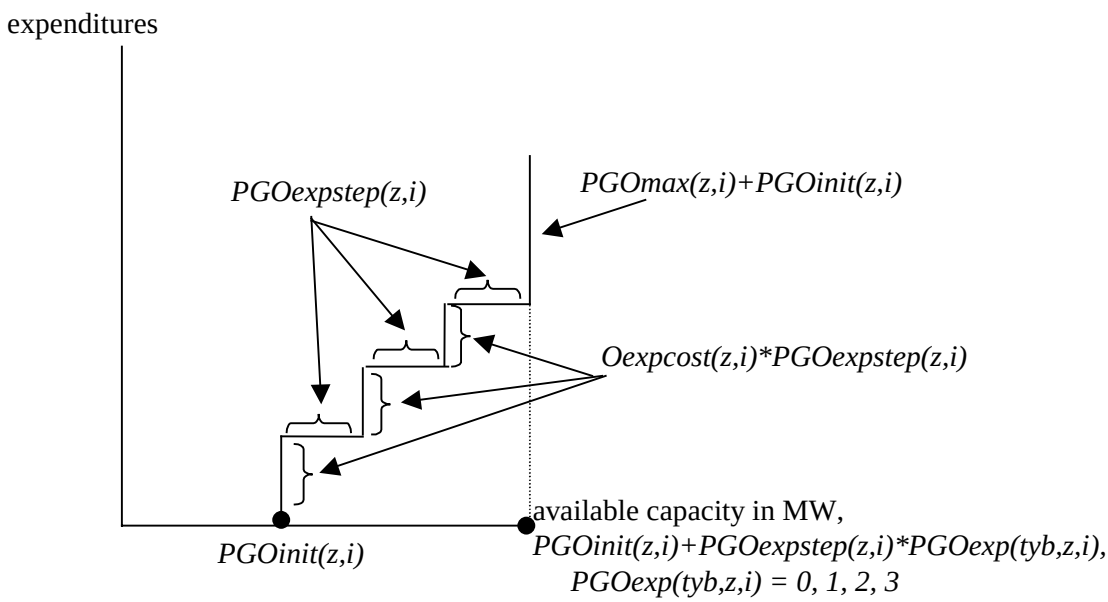
The user can select two options for modeling capacity expansion at existing sites - treating *PGOexp(tyb,z,i)* as a continuous variable, (e.g., fractional units are acceptable) or as an integer variable which allows only integer multiples of the given size plant.

Figures 3.1a and 3.1b show the two possible ways of modeling capacity expansion at an already developed site. Both represent the initial capacity by the parameter *PGOinit(z,i)*, and both have upper bounds on the total expansion possible at a site *PGOmax(z,i)*, tabled in Appendix IV, Section 14. They differ in the way the expansion is handled.

Figure 3.1a plots available capacity at site *i* as a function of the dollars expended on new capacity, if it is assumed that additional capacity is available in multiples of fixed

plant sizes, indicated in Figure 3.1a as the step width, *PGOexpstep(z,i)*, tabled in Appendix IV, Section 4.

Figure 3.1a: Expansion as Fixed Multiples of a Given Size, e.g., *PGOexp(tyb,z,i) = 0,1,2,3*, and *MW installed = PGOexp(tyb,z,i) * PGOexpstep(z,i)*, up to the maximum.



Note that the total cost of installing additional capacity is

$$\underbrace{Oexpcost(z,i) + PGOexpstep(z,i)}_{\text{cost per step}} * \underbrace{PGOexp(tyb,z,i)}_{\text{number of steps}}$$

The cost of a single integer expansion - the step height - is given by the product of the parameter *Oexpcost(z,i)*, in Section 2 of Appendix IV -which gives the cost/MW - times *PGOexpstep(z,i)*, the step width, in MW. If the integer decision variable *PGOexp(tyb,z,i)* is defined as the number of units of size *PGOexpstep(z,i)* (the step width) installed in year *ty*, then it must be true that capacity utilization in *ty* at site *i* cannot exceed the initial capacity plus the cumulative expansion up to and including that made in year *ty* e.g.; (ignoring decay)

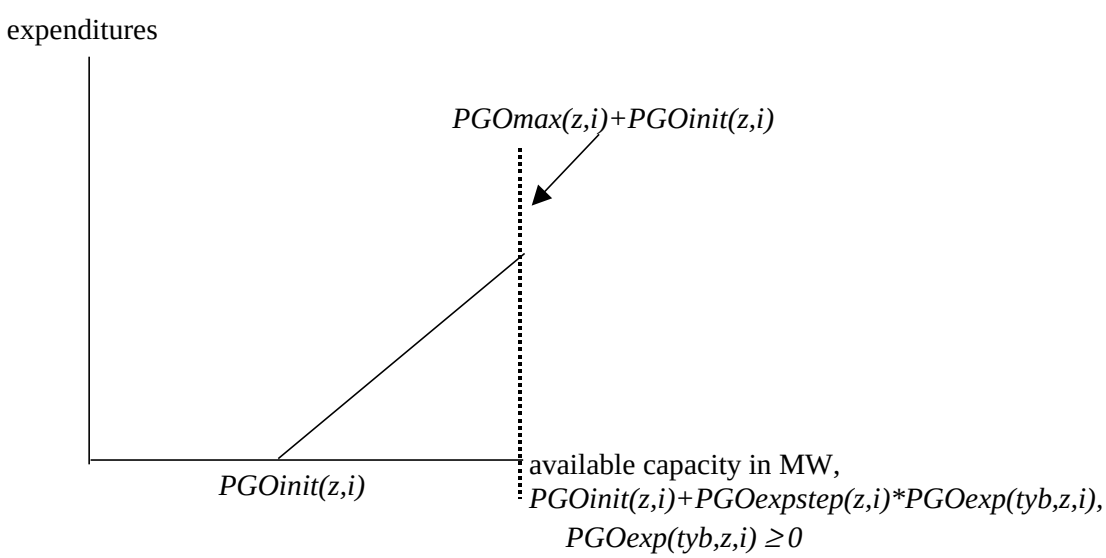
$$PG(ty,ts,td,th,z,i) \leq$$
$$\left\{ PGOinit(z,i) + \sum_{tyb=1}^{ty} PGOexpstep(z,i) PGOexp(tyb,z,i) \right\} * derate\ for\ day\ type.$$
$$PGOexp(ty,z,i) = 0,1,2,3...$$

Further, the total expansion over all periods at site *i* cannot exceed the maximum additional capacity allowed, *PGOmax(z,i)*, in Section 14 of Appendix IV; - e.g., letting *T* be the last year of the horizon;

$$\sum_{tyb=1}^{T} PGOexpstep(z,i)\, PGOexp(tyb,z,i) \leq PGOmax(z,i)$$

The GAMS notation equation *CON10* is in Section 22 of Appendix VII.

Figure 3.1b Expansion as a Continuous Variable, e.g., *PGOexp(tyb,z,i)* ≥ 0, and *MW installed = PGOexpstep(tyb,z,i) * PGOexp(tyb,z,i)* ≥ 0 up to the maximum.



Alternatively, Figure 3.1b assumes expansion to be a continuous, rather than in fixed multiples of a given plant size. Here it is assumed that there are enough variations in the size of the units available in the market place to allow the expansion variable to be

considered continuous, and then rounded off by the user to the nearest size available for the technology in question. If this assumption is made, the utilization equation is identical in form to the preceding equation, except the variable *PGOexp(tyb,z,i)* is declared positive, rather than integer.

Again, note that the total cost of a given expansion is *Oexpcost(z,i)* * *PGOexpstep(z,i)* * *PGOexp(t,z,i)* except *PGOexp(ty,z,i)* can take on fractional values.

The generic capacity constraint for existing thermal units is (again ignoring decay):

$$PG(ty,ts,td,th,z,i) \le \left\{ PGOinit(z,i) + \sum_{tyb=1}^{ty} PGOexpstep(z,i)\, PGOexp(tyb,z,i) \right\} derate\ by\ day\ type,$$

$$PGOexp(ty,z,i) \ \ge 0$$

The choice of which to use is up to the user. There is no “free lunch” in modeling; while assuming capacity expansion is possible only by adding multiple units of a given size is closer to reality, the user will pay a substantial run time penalty if that route is taken.

Conversely, if the continuous approach is taken, while run times are dramatically reduced, the error caused by rounding off to the nearest available unit size may cause problems.

Because of construction time lags, no construction is allowed in the initial period 1 - e.g., *PGOexp(“1”,z,i) = 0*.

The capacity constraint equation needs to reflect the gradual decay in generating capability as the units age, utilizing the modeling method described previously. Letting *DecayPGO* be the annual decay rate appropriate for each old thermal site (default values tabled in Appendix VI, Section 13) and recalling that periods can have a user specified number of years *n* per period, we have the following modification of the generic capacity constraint for old thermal plants:

$$PG(ty,ts,td,th,z,i) \le \Big\{ PGOinit(z,i)(1\text{-}DecayPGO)^{n(ty)} + \sum_{tyb=1}^{ty} PGO\exp step(z,i)PGO\exp(tyb,z,i)(1\text{-}DecayPGO)^{n(ty-\tau)} \Big\} * derate\ for\ day\ type$$

where *n(ty)* and *n(ty-τ)* adjust the exponent to account for the differing period lengths. The capacity constraints for the old thermal plants are found in Sections 21 and 22 of Appendix VII; equations *CON9a*, *CON9b*, *CON9c* and *CON9d*.

Finally, the forced and unforced outage rate notation for old units is *FORPGO(z,i)* and *UFORPGO(z,i)* respectively (default values in Appendix VI, Sections 2 and 10).

According to the latest SAPP list of approved optional projects, there are no plans to expand any of the existing thermal plants listed in Table 1.1. Consequently, the default data in Table *PGOmax(z,i)* are set at 0. If users wish to allow expansion, rather than consider only new site construction, they can do so by entering the maximum expansion amount in Table *PGOmax(z,i)* (Appendix IV, Section 14), the expansion step size *PGOexpstep(z,i)* (Appendix IV, Section 4), as well as the appropriate expansion cost/MW in Table *Oexpcost(z,i)* (Appendix IV, Section 2), and the model will automatically consider these options in the optimization.

Since there are no existing thermal expansion options in the model, there are no Before/At/After construction constraints created for existing thermal sites. If users wish to use this feature, the expansion should be entered as a new thermal project, which has those options.

Existing thermal plants can be decommissioned by entering the period when decommissioning is to take place in Table *Fdecom(z,i)* in Section 19 of Appendix IV. The characteristics of these plants can be altered by the user up to the date of decommissioning. Maximum and minimum generation constraints on old thermal units can be set by entering the desired load factor (decimal fraction of installed capacity) in Tables *PGmin(z,i)* and *PGmax(z,i)*, also found in Section 19 of Appendix IV.

3.2 Old Hydro Sites

The same constraints apply to existing hydro generation - that generation in year *ty*, in any period - the variable *H(ty,ts,td,th,z,ih)* -- cannot exceed the sum of properly derated (Table *DecayHO* in Appendix VI, Section 13) initial capacity -- *HOinit(z,ih)* (values in Appendix V, Section 4) plus new capacity installed by year *ty*, which is equal to the sum up to and including year *ty* of *HOVexp(ty,z,ih)*, a continuous or integer decision variable, times *HOexpstep(z,ih)*, (tabled in Appendix V, Section 5), for example. The model derates hydro capacity only by the forced outage rates *FORoh(z,ih)* (tabled in Appendix V, Section 8) since the model assumes preventive maintenance takes place when the reservoir volume falls below it's minimum level described below. Thus, only one capacity constraint is needed for all day types:

$$H(ty,ts,td,th,z,ih) \le \Big[HOinit(z,ih)(1-DecayHO)^{n(ty)} + \sum_{tye=1}^{ty} HOVexp(tye,z,ih) HOexpstep(z,ih)(1-DecayHO)^{n(ty-tye)} \Big] \big(1-FOROH(z,ih)\big)$$

with *HOVexp(ty,z,ih)* declared a positive or integer variable, depending on user preferences.

Again, *HOVexp("1",z,ih) = 0*. The capacity equations for the existing hydro plants, Equation *HOmw* are found in Appendix VII, Section 10.

Further, the upper limit on total additions at a site must be respected. Table *HOVmax(z,ih)* in Appendix V, Section 4 lists these upper limits on capacity additions:

$$\sum_{ty=1}^{T} HOVexp(ty,z,ih)\, HOexpstep(z,ih) \le HOVmax(z,ih)$$

or in GAMS format, equation *HOlimit* is shown in Appendix VII, Section 10.

Additional capacity constraints are needed for the hydro units to reflect the fact that the water supply, and hence the amount of electricity, which can be generated in a season, is limited by the reservoir's capacity and inflow.

This is done by a constraint that limits yearly generation from old and new sites to be limited to the kWh equivalent of the yearly volume of water allowed to be withdrawn from the reservoir. This parameter *HOLF(z,ih)* for old dams and *HNLF(z,nh)* for new is

determined by the user by entering the selected MWh amounts in the appropriate tables. Table 3.2 below gives these generation limits now in the model for existing dams. They can also be found in Section 6 of Appendix V. Users can adjust the kWh equivalent to reflect drought conditions in any period by specifying the parameter *fdrought(ty,z)*, in Section 1 of Appendix VIII, to be between zero and one. (Default settings now in the model are all 1 – e.g., no drought.) Thus, the yearly MWh constraint for existing dams is:

$$\sum_{ts,td,th} H(ty,ts,td,th,z,ih)\,Mseason(ts)\,Mday(td)\,Mtod(th) \leq fdrought(ty,z)\,HOLF(z,ih)$$

where *Mseason*, *Mday*, and *Mtod* are the proper season, day, and hour weights, which insure the left hand side of the equation, accounts for all 8760 hours of dam operations. This equation called Equation *MWhODam*, is found in Section 10 of Appendix VII.

A distinction needs to be made between the above constraint, equation *MWhODam*, and the constraint mentioned in Chapter 2, Section 2.2, which limits the range of capacity utilization by old hydro sites by specifying minimum and maximum load factors. While both specify a maximum MWh/year that can be obtained, the equation *MWhODam* is a physical constraint, set by the volume of water stored in the reservoir. The maximum capacity utilization constraints are discretionary constraints set by users arising from rules of operation of the SAPP system (e.g., “at no time will the SAPP plan involve less than x%, or more than y%, utilization of the installed capacity of site “z””).

Care must be taken in the use of the discretionary constraints to avoid infeasibilities that might arise if the minimum capacity utilization load factor resulted in a yearly minimum MWh constraint that exceeded the physical constraint set by the volume of water available.

Table 3.2 *HOLF(z,ih)*, Annual Generation Limit for Existing Dams (MWh per year)

	STAT1		STAT2		STAT3	
	Name	Generation Output	Name	Generation Output	Name	Generation Output
Ang	Cambambe I	690000	Matela	165000	Mabubas 1&2	58000
Bot						
Les	Muela	630720				
Mwi	Nkula A & B	530603	Tedzani 1, 2, 3	570248	Kapichiri A I&II	1121280
NMoz	*Hydro Cahora Bassa N.	1.66e7	Chicamba	14000	Mavuzi	142000
SMoz	Corumana	15000				
Nam	Ruacana	1090620				
SSA	** Gariep	2207520	** Vanderkloof	1927200		
Swz	Old Hydro 1	170820				
DRC	*** Inga 1, 2	7665000	*** Nseke	993384	*** Nzilo	425736
Taz	Kidatu	1240000	Mtera	626300	Pangani	443700
Zam	Kariba North (Ext.)	2500000	Kafue	4920000	Victoria Falls	700000
Zim	** Kariba South (Ext.)	2500000				

	STAT4		STAT5		STAT6	
	Name	Generation Output	Name	Generation Output	Name	Generation Output
Ang	Lomaum 1,2,4	157000	Biopio	23000	Capanda II	2200000
Bot						
Les						
Mwi						
NMoz						
SMoz						
Nam						
SSA						
Swz						
DRC	*** Mwadingusha	269633	*** Koni	110376	Zongo	236520
Taz	Hale	154800	N-Mungu	58700	Kihansi	1085364
Zam						
Zim						

*Assuming a 50% capacity factor. ** Data did not directly supply Generation Output MWh information. Data supplied is provided by the formula: Total Capacity*8760*40% *** 100% Capacity Factor – e.g., generation limit = Total MW capacity * 8760

According to the latest data provided by SAPP, the only existing hydro sites now being considered for expansion are one in DRC (Inga 3), one in Malawi (Kapichiri II), one in Zambia (Kariba N.) and one in Zimbabwe (Kariba S.).

Thus, the default data in the model allow only expansions of these sites to be considered. If, at a later time, users wish to allow expansion of other sites to be considered, they may do so by entering the values for such expansions in Tables *HOVmax*, *HOexpstep*, and *HOVcost*.

Constraints that require the hydro expansion project to be started or finished before or during a given period, or not be started until after a given period, can be found in Section 9 of Appendix V.

Tables *AtHO(z,ih)*, *BefHO(z,ih)* and *AftHO(z,ih)* require that one expansion step be completed at the period entered in Table *AtHO(z,ih)*, one expansion step be completed before the period entered in Table *BefHO(i,n)*, and one expansion step not be completed until after the period entered in Table *AftHO(z,ih)*. (The optimization controls any further expansion.)

Tables *AtHallO(z,ih)* and *BefHalOl(z,ih)* require that the maximum expansion allowed take place at or before the periods entered in these tables. In cases where there is only one expansion step in the expansion plan, both sets of constraints mean the same thing.

Constraints that set minimum and maximum load factors for old hydro projects (expressed as a decimal fraction of existing capacity) can be found in Table *minH(z,ih)* and *maxH(z,ih*) in Section 9 of Appendix V.

Old hydro units can be decommissioned by entering the period of decommissioning in Table *FdecomH(z,ih)*, which is also found in Section 9 of Appendix V.

3.3 New Thermal Plants

A distinction needs to be made between new sites, new plants, and new units before proceeding. Each region has a number of potential new generation <u>sites</u> that, if economic, can be developed by construction of new generating <u>plants</u> of varying types. The capacity of these <u>plants</u>, once the initial <u>site</u> development expense has been made, can be added to by investment in additional generating <u>units</u>. Thermal technologies available in the model include simple combustion turbines (*CT*), combined cycle turbines (*CC*), small coal (*SC*) plants, and large coal (*LC*) plants. Individual plant characteristics are specified by entering this technical and economic data into the appropriate tables in the model. Once this data (and a plant name) is entered into the model, the project will be

automatically incorporated into the optimization as an option. To be included, users must specify each of the following characteristics:

- The initial capacity in MW of the first unit, and its fixed cost (Tables *PGN__(.)* and *FG__(.)*.
- The step size in MW of any additional units to be considered for installation at the plant, and the expansion cost/MW (Tables *__expstep(.)* and *__expcost(.)*).
- The maximum possible additions at each plant in MW (Table *PGN__max(.)*).
- The heat rate (million BTU/MWh), fuel cost ($/million BTU), and fuel cost escalation rate (fraction/yr) (Tables *HRN__(.)*, *fpN__(.)*, and *fpescN__(.)*).
- Fixed *O&M* ($/MW/yr – expansion of old, and new plants only) and variable *O&M* ($/MWh – all plants) (Tables *FixOM__(.)* and *OM__(.)*).
- The yearly capital recovery factor, a function of the cost of capital and expected plant life (Tables *crf__(.)*).
- Forced and unforced outage rates (fraction/yr) (Tables *FORN__(.)* and *UFORN__(.)*).
- Capacity decay rates (fraction/yr) (scalar *DecayN__*).

Tables for all but the last two characteristics are found in Appendix IV; the last two are in Appendix VI.

The engineering and economic data for the plants currently proposed by SAPP, listed in Table 1.3, which are entered in the database, are based on data provided by the SAPP GPWG. Additional plants, up to eight per country, can be added to this list by simply entering the necessary data in the appropriate tables described below.

For purposes of dispatch, all the units of a given plant type are available for joint dispatch - e.g., *CC* plant generation is constrained only by the total installed *CC* plant capacity at a given site.

The four plant types can be broken down into two categories; those with scale economies, and those without. Plant types which have no scale economies - combustion turbines and small coal units - have the characteristic that if additional capacity of this

type is needed beyond the initial capacity, only replicas of the initial units which have identical costs and performance can be added to increase capacity.

The second type of new plant - large coal plants and combined cycle plants - are assumed to exhibit economies of scale, in that after installation of their initial capacity at a fixed cost to cover site preparation, each can add additional capacity in the form of units with lower capital cost/MW.

In cases where the need for additional capacity is not recurring and/or interest rates are high, small plants with no provision for growth are the preferred choice. When the need for additional capacity is immediate and continuing and/or interest rates are low, plants with high initial development costs, but much lower expansion costs, will be preferred.

In the case of *CT* and *SC* plants, additional capacity can be added only by constructing replicas of the original plants.

In the case of large *CC* and *LC* plants, capacity can be increased in two ways - adding new plants, as is the situation for *CT* and *SC*, and adding cheaper units to plant sites already developed.

In all cases, only one plant per site can be built – see *CCsum* and *LCsum* in Section 5 of Appendix VII.

3.4 Combustion Turbines and Small Coal Units

The technical parameters which enter into the capacity constraints of the two technologies - unit capacity *NTexpstep(z,ni)* and *NSCexpstep(z,ni)*, forced and unforced outage rates *FORNT(z,ni)*, *UFORNT(z,ni)*, *FORNSC(z,ni)*, and *UFORNSC(z,ni)* - are given in Appendix IV, Sections 5 and 6 (expansion steps) and Appendix VI, Sections 2, 4, 5, and 7. Users can input their own values to reflect specific site parameters, or add additional plants by entering the appropriate data in the tables.

Since the small gas turbines modeled here are not designed to be operated continuously, but only during periods of peak demand, their use in the model is restricted

to the peak periods in the model, here taken to be the 4-peak hours/day, or 1460 hours/year.

The decision to build a plant at site *ni* is reflected in the value of the decision variable *PGNTexp(ty,z,ni)* for the *CT*, and *PGNSCexp(ty,z,ni)* for the small coal unit. The model allows the user to specify how the construction of new *CT* and *SC* plants are to be modeled - as a continuous variable, or in fixed sizes. In both cases, the fixed capacities (step widths) are specified in Tables *NTexpstep* and *NSCexpstep*, found in Sections 5 and 6 of Appendix IV and the cost per MW for these steps – *NTexpcost* and *NSCexpcost* – are found in Sections 3 and 4 of Appendix IV. The choice of continuous versus fixed expansion is made by declaring the decision variables *PGNTexp(ty,z,ni)* and *PGNSCexp(ty,z,ni)* either continuous, or integer. Figures 3.2a and 3.2b show the plot of available capacity and cost if fixed multiples (3.2a) or continuous (3.2b) expansion is chosen, dropping the *T* and *SC* index from the variable names.

Figure 3.2a: Expansion as Fixed Multiples of a Given Plant Size; *PGN_ _exp(ty,z,ni) = 0,1,2,3,...*

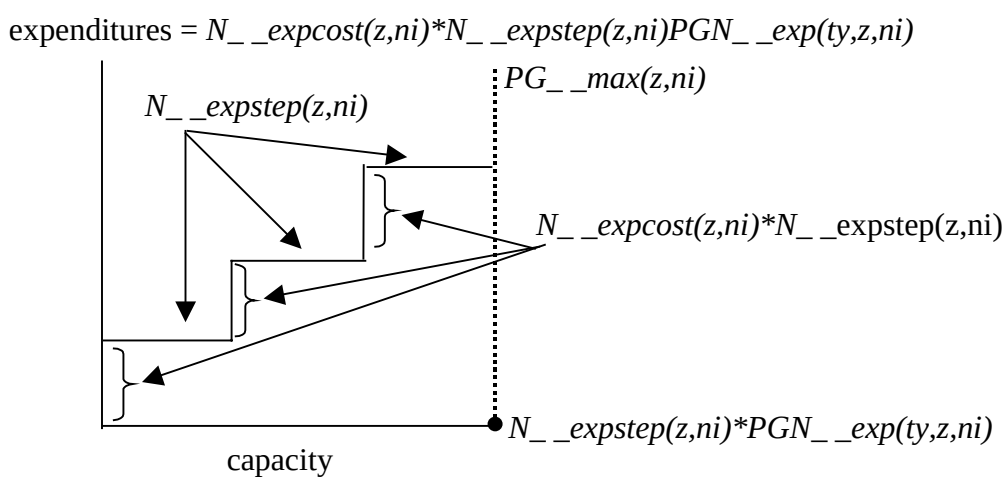
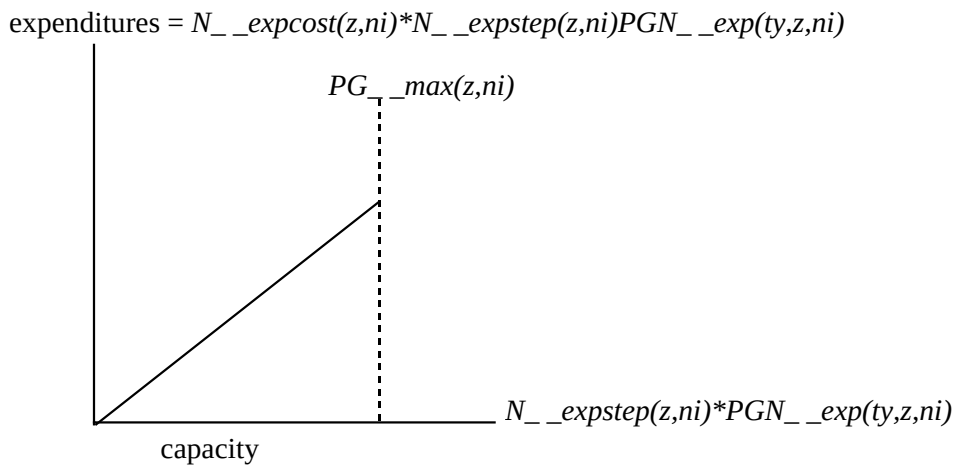


Figure 3.2b: Expansion as a Continuous Variable; *PGN_ _exp(ty,z,ni)* ≥ 0



If *NTexpstep(z,ni)* and *NSCexpstep(z,ni)* are the fixed capacities of these units, and *DecayNT* and *DecayNSC*, found in Section 13 of Appendix VI, the aging factors, the capacity constraints are of a simple form:

$$PGNT\left(ty,ts,td,th,z,ni\right) \leq \left[\left(\sum_{\tau=1}^{ty} PGNTexp\left(\tau,z,ni\right) NTexpstep\left(z,ni\right)\right)\left(1 - DecayNT\right)^{n(ty-\tau)}\right] Derate\ \ for\ day\ type$$

with *PGNTexp(ty,z,ni)* either positive or integer.

Further, the upper limit on site capacity is:

$$\sum_{\tau=1}^{T} PGNTexp\left(\tau,z,ni\right) NTexpstep\left(z,ni\right) \leq PGNTmax\left(z,ni\right)$$

For the small coal units:

$$PGNSC(ty,ts,td,th,z,ni) \leq \left[\left(\sum_{\tau=1}^{ty} PGNSCexp(\tau,z,ni)\, NSCexpstep(z,ni)\right)(1 - DecayNSC)^{n(ty-\tau)}\right] Dereate \;\; for\ day\ type$$

with *PGNSCexp(ty,z,ni)* either positive or integer, and

$$\sum_{\tau=1}^{T} PGNSCexp(\tau,z,ni)\, NSCexpstep(z,ni) \leq PGNSCmax(z,ni)$$

Tables *PGNTmax(z,ni)* and *PGNSCmax(z,ni)* give the maximum expansion per site allowed for these two technologies; they are found in Sections 14 and 15 of Appendix IV.

Constraints that require the construction of new SC and CT plants before, after, and at a particular period can be founding Section 19 of Appendix IV.

Tables *AtSC(z,ni)*, *BefSC(z,ni)*, and *AftSC(z,ni)* require that one expansion step be completed at the period entered in each of the tables. Likewise, Tables *AtSCall(z,ni)* and *BefSCall(z,ni)* require that the maximum expansion allowed take place at or before the period entered in these tables. For those projects that have only one expansion step, both constraint sets say the same thing.

Constraints that set minimum and maximum load factors for SC plants (expressed as a decimal fraction of existing capacity) can be found in Tables *minSC(z,ni)* and *maxSC(z,ni)* in Section 19 of Appendix IV.

The same set of constraints is presented for CT in Section 19, immediately following those for SC plants.

The equations for CT and SC units in GAMS format for each of the 6-day types and the upper limits on capacity expansion are given in equations *CON1a-d (CT)*,

CON3a-d (SC), *CON7 (CT)*, and *CON8 (SC)* in Appendix VII, Sections 17, 18, 19, 20, and 21.

3.5 Large Combined Cycle and Large Coal Plants

Figures 3.3a and 3.3b show two possible ways of modeling total equipment cost as a function of capacity installed at new sites, when scale economies are present, as is assumed for combined cycle and large coal plants. Both assume that the fixed cost of installing the new initial capacity is substantial, including as it does the cost of site acquisition and preparation, connections to the grid and fuel handling equipment, all oversized to allow add on capacity at lower incremental cost. These initial fixed costs are given in Table *FGCC(z,ni)* for CC, and Table *FGLC(z,ni)* for LC in Section 1 of Appendix IV. Both assume the initial investment is a fixed cost, in that it must be paid in its entirety, or not at all. Thus, the decision to develop the initial site and install the initial capacity is always controlled by the binary variables *YLC(ty,z,ni)* for large coal, and *YCC(ty,z,ni)* for combined cycle plants. Both also have an upper limit on the add-on capacity, *PGNCCmax(z,ni)* and *PGNLCmax(z,ni)* in Appendix IV, Sections 15 and 16, set by the size of the initial site. As in the case of old capacity expansion, the two figures differ in their handling of the capacity expansion itself, the variables *PGNLCexp(tyb,z,ni)* and *PGNCCexp(tyb,z,ni)*. Figure 3.3a assumes that additions are available only in fixed sizes - e.g., *x* MW units - while Figure 3.3b assumes that there are enough variations in size in the units which can be purchased to allow the expansion variable to be continuous, and then rounded off to the nearest size available for that technology.

Thus for the combined cycle plant, the approach used in Figure 3.3a would add capacity in multiples of a specified number of MW each, while the approach in Figure 3.3b would add continuous MW capacity, which would then be rounded off to the nearest purchasable mix of units.

<u>Option #1:</u> Fixed increments of additional capacity. (Figure 3.3a)

In this formulation, capacity additions at a given site *ni* in a given period *ty* are the product of a parameter *NCCexpstep(z,ni)* for *CC* (found in Section 5 of Appendix IV),

NLCexpstep(z,ni) for *LC* (found in Section 6 of Appendix IV) which gives the user specified MW size of the units to be added, times the integer variable *PGNCCexp(tyb,z,ni)* for *CC*, *PGNLCexp(ty,z,ni)* for *LC*, which gives the number of units added in period *ty*. For example, if the expansion step is 250 MW, and the binary variable is chosen 1 in period *ty* then (1.)(250) = 250 MW would be added in the period.

Cost per MW for these step sizes are given by the user-specified parameters *NCCexpcost(z,ni)* and *NLCexpcost(z,ni)*, values are found in these tables in Sections 3 and 4 of Appendix IV.

<u>Option #2:</u> Continuously variable capacity. (Figure 3.3b)

In this formulation, the parameters *NCCexpstep(z,ni)* and *NLCexpstep(z,ni)* are kept the same, but the variables *PGNCCexp(tyb,z,ni)* and *PGNLCexp(tyb,z,ni)* are declared positive, continuous variables, rather than integer variables. In the example, if the continuous variable was chosen in period *ty* to be 1.5, then (1.5)(250) = 375 MW would be added in the period.

As before, the choice is up to the user. While the "lumpy" nature of the additions is closer to reality, the user will pay a substantial run time penalty, if that route is taken.

In both cases, the decision to develop and build the initial site is separate from the decision to expand the capacity of the site, once it is built. The decision to spend *FGCC(z,ni)* or *FGLC(z,ni)* (found in Sections 1 and 2 of Appendix IV) to initially develop the site and install *PGNCCinit(z,ni)* or *PGNLCinit(z,ni)* MW of capacity (found in Section 1 of Appendix IV) are determined in the model by the value of the binary variables *YCC(ty,z,ni)* or *YLC(ty,z,ni)* are 0, then construction did not take place in period *ty*. (Note this does not imply the site had not been developed prior to period *ty*.)

Figure 3.3a New plant Initial Construction, with Expansion as Fixed Multiples of a Given Size, e.g., *PGN_ _exp(tyb,z,ni) = 0,1,2,3...*

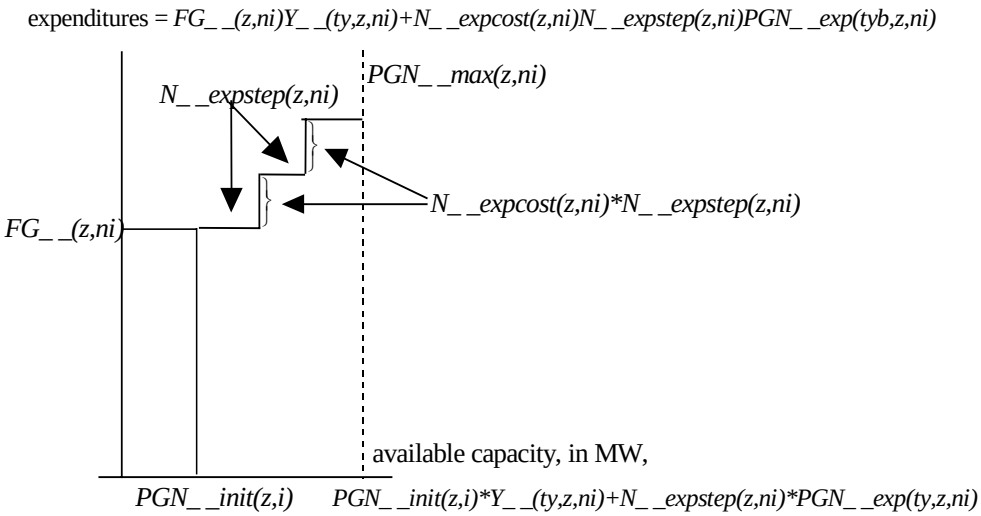
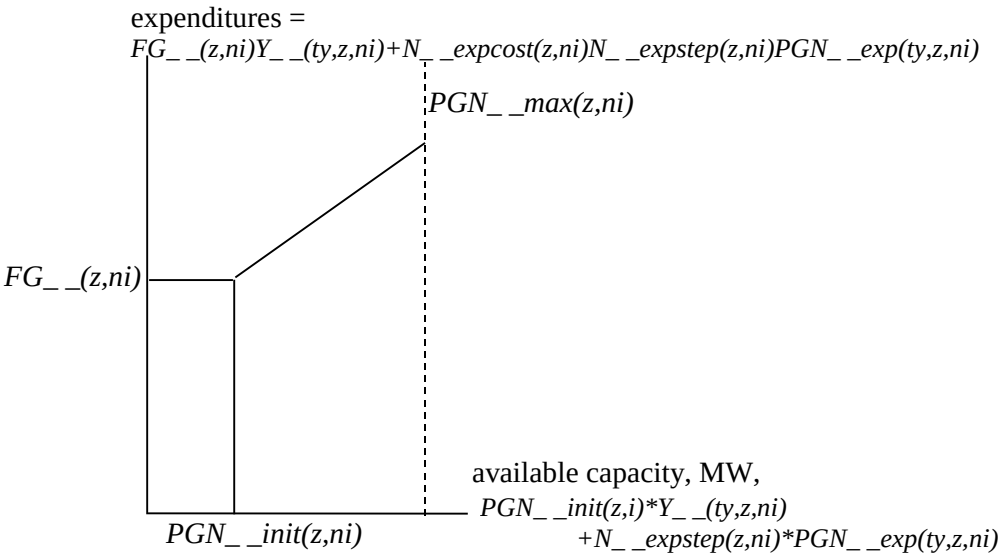


Figure 3.3b New Plant Initial Construction, with Continuous Expansion, e.g., *PGN_ _exp(ty,z,ni) ≥ 0*



Both formulations require that however capacity expansion is modeled, total expansion cannot exceed the upper limit on installed capacity at a site *PGNCCmax(z,ni)* or *PGNLCmax(z,ni)*, in Appendix IV, Section 15 and 16.

Each period, at most one new plant of a given type can be added to a new site.

As stated previously, additional generating units can be added to the new sites at any time, as long as the capacity of the plants to add units is not exceeded. Each new plant at a site, then, has three parameters critical to the capacity constraints:

- *PGNCCinit(z,ni)* and *PGNLCinit(z,ni)*, the initial plant capacity installed when the new site is first constructed (Section 1 of Appendix IV).
- *NCCexpstep(z,ni)* and *NLCexpstep(z,ni)*, the MW size of units added to a plant, once one is built at site *ni* (Sections 5 and 6 of Appendix IV).
- *PGNCCmax(z,ni)* and *PGNLCmax(z,ni)*, the upper limit on the total MW capacity, which can be added by additional units to the plant (Sections 15 and 16 of Appendix IV).

The decision variables *PGNCCexp(tyb,z,ni)* and *PGNLCexp(tyb,z,ni)* are either integer variables, if a fixed step size is selected, or continuous variables. Users can specify all period 1 variables to be zero, to reflect the fact that the required construction lead-time is too long to allow new plant to be available in the first period.

Assuming all units in all plants at a site will be dispatched collectively - and *DecayNCC* and *DecayNLC* are the aging factors (tabled in Appendix VI, Section 13), the capacity constraints for both the *CC* and *LC* technologies can be stated.

For *CC*, we have:

$$PGNCC(ty,ts,td,th,z,ni) \le \left[PGNCCinit(z,ni) \sum_{tya=1}^{ty} YCC(tya,z,ni)(1-DecayNCC)^{n(ty-tya)} + \sum_{tyb=1}^{ty} PGNCC\exp(tyb,z,ni)NCC\exp step(z,ni)(1-DecayNCC)^{n(ty-tyb)} \right] derate\ for\ day\ type$$

$$where\ YCC(tya,z,ni) = \{0,1\},\ PGNCC\exp(tyb,z,ni)\ either \ge 0\ or\ = 0,1,2,3,etc.$$

e.g., in period *ty*, the dispatch limit at site *ni* is equal to the number of plants built on the site as of period *ty* times their initial capacity plus the capacity added to these plants as of *ty*. Further,

$$\sum_{tya=1}^{ty} NCCexpstep(z,ni)\,PGNCCexp(tya,z,ni) \le \sum_{tya=1}^{ty} YCC(tya,z,ni)\,PGNCCmax(z,ni)$$

e.g., the upper limit on capacity added to all plants built at site *ni* up to *ty* is set by the number of plants built as of period *ty* times the upper limit on the capacity which can be added.

The limit of large coal generation at site *ni* in country *z* is:

$$PGNLC(ty,ts,td,th,z,ni) \le \Bigg[PGNLCinit(z,ni)\sum_{tya=1}^{ty} YLC(tya,z,ni)(1-DecayNLC)^{n(ty-tya)}$$
$$+\sum_{tyb=1}^{ty} PGNLC\exp(tyb,z,ni)NLC\exp step(z,ni)(1-DecayNLC)^{n(ty-tyb)}\Bigg] derate\ for\ day\ type$$
$$where\ YLC(tya,z,ni)=\{0,1\},\ PGNLC\exp(tyb,z,ni)\ either \ge 0\ or\ =0,1,2,3,etc.$$

and

$$\sum_{tya=1}^{ty} PGNLCexp(tya,z,ni)\,NLCexpstep(z,ni) \le \sum_{tya=1}^{ty} YLC(tya,z,ni)\,PGNLCmax(z,ni)$$

As in the case of the capacity constraints on existing sites, the installed capacities of new sites need to be derated for new plant forced outage rates (Tables *FORNCC(z,ni)* and *FORNLC(z,ni)* are found in Sections 3 and 5 of Appendix VI) during peak days, and both forced and unforced outage rates (Tables *UFORNCC(z,ni)* and *UFORNLC(z,ni)* are found in Sections 6 and 7 of Appendix VI) and for average days. The equations, which enforce the six-day type capacity constraints for new sites are given in GAMS format, followed by the expansion limit constraints, in equations *CON2a-d (CC)*, *CON4a-d (LC)*, *CON5 (CC)*, and *CON6 (LC)*, found in Appendix VII, Sections 18, 19, 20, and 21.

Constraints that require the construction of new CC and LC plants before, after, and at a particular period can be founding Section 19 of Appendix IV.

Tables *AtCC(z,ni)*, *BefCC(z,ni)*, and *AftCC(z,ni)* require that one expansion step be completed at the period entered in each of the tables. Likewise, Tables *AtCCall(z,ni)* and *BefCCall(z,ni)* require that the maximum expansion allowed take place at or before the period entered in these tables. For those projects that have only one expansion step, both constraint sets say the same thing.

Constraints that set minimum and maximum load factors for CC plants (expressed as a decimal fraction of existing capacity) can be found in Tables *minCC(z,ni)* and *maxCC(z,ni)* in Section 19 of Appendix IV.

The same set of constraints is presented for LC in Section 19 preceding the tables for CC.

3.6 The Current Treatment of SAPP Approved New Thermal Plants

As of May 2000, the following new thermal plant construction projects were proposed for consideration (Table 1.3). All data can be found in the appropriate tables in Appendix IV – expansion steps in Sections 5 and 6; maximum expansion in Sections 14 and 15; expansion costs/MW in Sections 3 and 4; fuel costs (Tables *fpNT*, *fpNCC*, *fpNSC*, and *fpNLC*) in Sections 7 and 8; heat rates in Sections 11, 12, and 13 (Tables *HRNT*, *HRNCC*, *HRNLC*, and *HRNSC*); and fuel cost escalation rates in Sections 9 and 10 (Tables *fpescNT*, *fpescNCC*, *fpescNLC*, and *fpescNSC*).

<u>Small Coal Projects</u>:

- Botswana's Moropule extension, decision variable *PGNSCexp(tyb,"BOT",1)* consisting of 2 units of 115 MW each, will be treated as a new small coal site, with the step size *NSCexpstep("BOT",1)* equal to 115 MW, *PGNSCmax("BOT",1)* equal to 230 MW, and the expansion cost/MW, *NSCexpcost("BOT",1)* equal to $850000/MW. Since no heat rates or fuel

costs have been provided, default values (11.60 million BTU/MWh, $.30/10^6 BTU) will be used.

- South Africa's Komati A site, (currently mothballed) decision variable *PGNSCexp(tyb,"NSA",1)* consisting of 9 units of 100 MW each, will have a step size *NSCexpstep("NSA",1)* equal to 100 MW, *PGNSCmax("NSA",1)* equal to 900, and a re-commission cost/MW, *NSCexpcost("NSA",1)* of $139400/MW. A heat rate of 12.88 million BTU/MWh and a fuel cost of $.67/10^6 BTU are used, taken from SAPP data. According to GPWG recommendations at 2000 Cape Town Workshop, the forced outage rate is set at 30%.
- South Africa's Grootvlei site, (currently mothballed) decision variable *PGNSCexp(tyb,"NSA",2)* consisting of 6 units of 190 MW each, will have a step size *NSCexpstep("NSA,2)*, equal to 190 MW, *PGNSCmax("NSA",2)* equal to 1140 MW, and a re-commissioning cost *NSCexpcost("NSA",2)* of $109000 /MW. A heat rate of 12.19 million BTU/MWh and a fuel cost of $.63/10^6 BTU are used, taken from SAPP data. Forced outage rate is set at 30%.
- South Africa's Komati B site, (currently mothballed) decision variable *PGNSCexp(tyb,"NSA",3)* consisting of four 110 MW units, will have a step size *NSCexpstep("NSA",3)* of 110 MW, *PGNSCmax("NSA",3)* equal to 440 MW, and a re-commissioning cost *NSCexpcost("NSA",3)* of $542000/MW. A heat rate of 12.186 million BTU/MWh, and a fuel cost of $.60/10^6 BTU are used taken from the SAPP data sheet. Forced outage rate is set at 30%.
- South Africa's Camden site, (currently mothballed) decision variable *PGNSCexp(tyb,"NSA",4)* consisting of 8 sites of 190 MW each , will have a step size *NSCexpstep("NSA",4)* of 190 MW, *PGNSCmax("NSA",4)* of 1520 MW, and a re-commissioning cost of $108000/MW. A heat rate of 12.19 million BTU/MWh and a fuel cost of $.70/10^6 BTU are used. Forced outage rate is set at 30%.

- South Africa's PB reactor site, decision variable *PGNSCexp(tyb,"NSA",5)* consisting of 1 unit of 1000 MW, will be treated in the model as a small coal unit, with a step size *PGNSCexpstep("NSA",5)* of 1000 MW, *PGNSCmax("NSA",5)* of 1000 MW, and an expansion cost *NSCexpcost("NSA",5)* of $844000/MW. A heat rate of 7.58 million BTU/MWh, and a fuel cost of $.45/$10^6$ BTU are used.
- Zimbabwe's Hwange 7 & 8, decision variable *PGNSCexp(tyb,"ZIM",2)* two units of 300 MW each, will have a step size of *NSCexpstep("ZIM",2)* of 300 MW, with *PGNSCmax("ZIM",2)* set at 600 MW, and *NSCexpcost("ZIM",2)* at $992000/MW. A heat rate of 9.57 million BTU/MWh and a fuel cost of $.51/$10^6$ BTU are used, taken from the SAPP data sheet.
- Zimbabwe's Gokwe North project, decision variable *PGNSCexp(tyb,"ZIM",3)*, 4 units of 321 MW each, will have a step size *NSCexpstep("ZIM",3)* of 321 MW, *PGNSCmax("ZIM",3)* of 1284 MW, and *NSCexpcost("ZIM",3)* of $860500/MW, a heat rate of 9.24 million BTU/MWh, and a fuel cost of $.88/$10^6$ BTU.
- Zambia's Lusaka project, is a single unit small coal plant of 100 MW with no plans for expansion. Thus, *PGNSCinit("ZAM",1)*, 100 MW and *PGNSCmax("ZAM",1)* are zero (no expansion possible). The fixed cost is $50e6. SAPP provided values of 16.152 million BTU/MWh for the heat rate and $5.54/$10^6$ BTU for fuel cost are used.
- Zambia's Maamba project, is a single unit small coal plant of 160 MW with no plans for expansion. Thus, *PGNSCinit("ZAM",2)*, 160 MW and *PGNSCmax("ZAM",2)* are zero (no expansion possible). Expansion costs are $231,000/MW. SAPP provided values of 13.398 million BTU/MWh for the heat rate and $1.53/$10^6$ BTU for fuel cost are used.

<u>Large Coal Projects</u> (one project)

- South Africa's Lekwe site, with 6 units of 659 MW each, will be treated as a large coal site, with an initial step and expansion step size of 659 MW. Since

this is a generic plant type capable of being constructed at many sites, the model allows up to 7 additional sites in NSA. Thus, *PGNLCinit("NSA",1)* and *NLCexpstep("NSA",1)* both will be 659 MW. *PGNLCmax("NSA",1)* will be 3295 MW, since the initial step is not counted as expansion. The initial site construction variable, *YLC(tya, "NSA",1)* will always be binary; the expansion variable *PGNLCexp(tyb,"NSA",1)* will always be integer.

Since the data furnished by SAPP does not distinguish between the initial and subsequent expansions, it will be assumed that the first step's cost - *FGLC("NSA",1)* - will be $1.014e9. The remaining 5 units would then cost $923000/MW to arrive at the same total cost of $4.056 billion dollars given in the SAPP data for the entire 3954 MW.

Heat rates of 9.89 million BTU/MWh and fuel costs of $.42/$10^6$ BTU given in the SAPP table will be used.

<u>Combined Cycle Projects</u> (2 projects)

- Namibia's Kudu site is a single unit combined cycle plant of 750 MW with no plans for expansion. Thus, *PGNCCinit("NAM",1)* is 750 MW, and *PGNCCmax("NAM",1)* is zero (no expansion possible). The decision variable *YCC(ty, "NAM",1)* is binary, and no replications are allowed. The fixed cost *FGCC("NAM",1)* is $335e6. Default values of 7.183 million BTU/MWh for the heat rate and $2.00/$10^6$ BTU for gas fuel cost are used in the absence of specific data.
- Tanzania's Ubungo project is a single unit combined cycle plant of 40 MW with no plans for expansion. Thus, *PGNCCinit("TAN",1)* is 40 MW, and *PGNCCmax("TAN",1)* is zero (no expansion possible). The decision variable *YCC(ty,"TAN",1)* is binary, and no replications are allowed. The cost/MW is $610,000. Default values of 9.0 million BTU/MWh for the heat rate and $2.56/$10^6$ BTU for gas fuel cost are used in the absence of specific data.

Gas Turbine Units (3 projects)

The list of SAPP approved optional projects currently includes the following gas turbine projects:

- Angola proposes to add one gas turbine of 25 MW. The step size *NTexpstep("ANG",1)* will be 25 MW, the maximum expansion *PGNTmax("ANG",1)* will be 0 MW, and the expansion cost *NTexpcost("ANG",1)* will be $300000/MW, as reported in the SAPP table. Heat rates of 11.25 million BTU/MWh and fuel costs of $6.766/10^6 BTU will be used, as reported in the SAPP tables.
- The other proposed large gas turbine in Angola is 60 MW. The step size *NTexpstep("ANG",2)* will be 60 MW, the maximum expansion *PGNTmax("ANG",2)* will be 0 MW, and the expansion cost *NTexpcost("ANG",2)* will be $300000/MW, as reported in the SAPP table. Heat rates of 11.25 million BTU/MWh and fuel costs of $7.89/10^6 BTU will be used, as reported in the SAPP tables.
- South Africa proposes to add 4 large gas turbines of 250 MW each. The step size *NTexpstep("SA",1)* will be 250 MW, the maximum expansion *PGNTmax("SA",1)* will be 1000 MW, and the expansion cost *NTexpcost("SA",1)* will be $324000/MW, as reported in the SAPP table. Heat rates of 9.478 million BTU/MWh and fuel costs of $6.19/10^6 BTU (!) will be used, as reported in the SAPP tables.

3.7 New Hydro Capacity Constraints

As in the case of thermal plants, while existing hydro plants economically can increase their capacity up to some limit, new plants at a site must pay the fixed cost of site preparation and construction - usually a much larger investment than thermal units, since it involves dam construction - before any expansion can take place. The model notation for production from new hydro facilities at site *nh* is *Hnew(ty,ts,td,th,z,nh)*; the initial capacity parameters for plants at site *nh*, Table *HNinit(z,nh)*, are in Section 1 of Appendix

V. Table *HNVmax(z,nh)* the maximum possible additional capacity, which can be installed at a site, is found in Section 2 of Appendix V. The decision variable *HNVexp(ty,z,nh)*, which indicates MW expansion in year *ty*, can be either an integer, or a continuous variable; step size is given in Table *HNexpstep(z,nh)*, which can be found in Section 3 of Appendix V.

The only difference between the old and new hydro constraints is the inclusion of the binary decision variable *Yh(ty,z,nh)* in the capacity constraints; as before, *Yh(ty,z,nh) = 1* if a plant is built in *ty*, 0 otherwise, except *Yh("yr1",z,nh) = 0*. Since preventive maintenance is assumed to take place when the reservoir is below minimum levels for generation, capacity is derated only by the forced outage rate *FORnh(z,nh)* given in Section 8 of Appendix V. Thus, letting *DecayHN* (Section 13, Appendix VI) be the aging factor, the new hydro capacity constraints are:

$$Hnew(ty,ts,td,th,z,nh) \le \left[HNinit(z,nh)\sum_{tye=1}^{ty} Yh(tye,z,nh)(1-DecayHN)^{n(ty-tye)} + \sum_{tye=1}^{ty} HNV\exp(tye,z,nh)HN\exp step(z,nh)(1-DecayHN)^{n(ty-tye)} \right]\left[1-FORNH(z,nh)\right]$$

where *Yh(ty,z,nh) = {0,1}*, *HNVexp(ty,z,nh)* either integer or positive.

Further,

$$\sum_{tye=1}^{ty} HNexpstep(z,nh)\,HNVexp(tye,z,nh) \le \sum_{tye=1}^{ty} Yh(tye,z,nh)\,HNVmax(z,nh)$$

for example, the upper limit on capacity additions must be respected, and no expansion can take place unless the initial plant is built $\left(\sum_{tye=1}^{ty} Yh(tye,z,nh) = 1\right)$; *HNVmax(z,ih)* is found in Appendix V, Section 2.

These constraints, in GAMS format, are equations *HNmw* and *HNmust*, as shown in Appendix VII, Section 9.

Finally, as in case of existing dams, there is a limit on the amount of water per year which can be used for generation, and hence a limit on the total MWh per year which can be generated, *HNLF(z,nh)* (Section 6, Appendix V). Hence (Equation *MWhNDam*, Appendix VII, Section 10), neglecting weights,

$$\sum_{td,th} Hnew(ty,ts,td,th,z,nh) \leq HNLF\left(z,nh\right)\ fdrought(ty)$$

As in the case of old dams, adding capacity is assumed to involve only adding water turbines, not increasing the storage capacity of the dams. Thus *HNVexp* does not enter into either of the above equations. Further, no decay is assumed in the ability of the structure to hold the design volume of water; therefore, no decay is entered in the right hand side of either Case A or Case B.

As in the case of existing dams, absent SAPP data on MWh available per period, it will be assumed that the facility can generate at its rated MW capacity 100% of the time.

Table 1.3 lists the new hydro projects now in the database.

Constraints that require a new hydro project to be started, or finished before or during a given period, or not be started until after a given period, can be found in Tables *AtHn(z,nh)*, *BefHn(z,nh)*, *AtHnall(z,nh)*, *BefHnall(z,nh)* and *AftHn(z,nh)* in Section 9 of Appendix V. Constraints that set minimum and maximum load factors (expressed as a decimal fraction of existing capacity) can be found in Tables *minHN(z,nh)* and *maxHN(z,nh)* in Section 9 of Appendix V.

3.8 Old Transmission Lines

Table 1.7 in Chapter 1 summarizes the basic data in the model for existing lines; the data are presented in Appendix III in matrix format in Table *PFOinit(z,zp)* (direction of flow from row name to column name) for the initial capacity (Section 1 of Appendix III), Table *FORICO(z,zp)* for forced outage rates (Appendix VI, Section 9), Table

PFOloss(z,zp) for line loss (Section 3 of Appendix III), and Table *PFOVmax(z,zp)* for the upper limit on expansion (Section 3).

Currently, with the exception of DC lines, entries are all symmetric, in that the same data appear for line characteristics in either direction. However, users may enter whatever values they wish regarding the directional line characteristics; for instance, if it is known that transfer capabilities differ depending on the flow direction, they may be entered in the table to indicate that flow capability from x to y differs from the flow capability from y to x. (The same holds for loss coefficients and forced outage rates.)

DC lines can be characterized as (a) lines that have positive capacity in the allowed flow direction, and 0 capacity in the counter flow direction and/or (b) lines that have finite line loss in the allowed flow direction, but infinite line loss (0.9999) in the counter flow direction. The tables now use only the second method – infinite line loss – to distinguish DC from AC lines, but users can use both or just 0 capacity, the effect is the same.

Using the familiar notation applied to existing transmission lines, power flow on a directed old link, *PF(ty,ts,td,th,z,zp)* the sum of firm and non-firm flows, must, in year *ty*, be less than or equal to existing capacity, *PFOinit(z,zp)* (tables in Appendix III, Section 1) plus any expansion up to year *ty*, derated by the forced outage rate for transmission *FORICO(z,zp)* (tabled in Appendix VI, Section 9), and adjusted for decay using *DecayPFO* (Section13, Appendix VI). (Unforced outages in transmission are considered insignificant.) Letting *PFOVexp(ty,z,zp)* be the continuous variable indicating the MW transmission capacity expanded in year *ty*, we have:

$$PF\left(ty,ts,td,th,z,zp\right) \le \Bigg[PFOinit\left(z,zp\right)\left(1-DecayPFO\right)^{n(ty)} + \sum_{tye=1}^{ty} PFOVexp\left(tye,z,zp\right)\left(1-DecayPFO\right)^{n(ty-tye)} \Bigg]\left(1-FORICO\left(z,zp\right)\right)$$

or, in GAMS notation, Equation *PFOmw*, as shown in Appendix VII, Section 12. (Note the assumption that transmission capacity can only be added continuously.)

Also, the upper limit on capacity expansion on existing lines, given in Table *PFOVmax(z,zp)* (in Appendix III, Section 3), must be respected. In GAMS format, equation *PFOlimit* is in Appendix VII, Section 12.

Expansion in the (*z,zp*) direction also expands capacity in the (*zp,z)* direction, for all lines as indicated in Equation *PFOdirect* in Section13 of Appendix VII. Thus, users do not have the option of using 0 capacity in the counter flow direction to characterize expansion of DC lines; the only option is the infinite loss option in the counter flow direction. (See for instance, Table *PFOloss(NSA,NMz)* = 0.999 in Section 3 of Appendix III.)

Constraints that set minimum and maximum load factors (expressed as a decimal fraction of existing capacity) for existing transmission lines can be found in Tables *minPFO(z,zp)* and *maxPFO(z,zp)* in Section 7 of Appendix III.

3.9 New Transmission Lines

Figure 1.3 lists the possible augmentations to current transmission capacity now being considered by SAPP members for the horizon. Transfer capabilities, forced outage rates, and other data are listed in Table 1.8.

The only change required for new lines is to add the binary variable, *Ypf(ty,z,zp)*, to the capacity constraint to insure that flow will not take place without the full initial line being built - e.g.,

$$PFnew(ty,ts,td,th,z,zp) \le \left[PFNinit(z,zp) \sum_{tya=1}^{ty} Ypf(tya,z,zp)(1-DecayPFN)^{n(ty-tya)} + \sum_{tya=1}^{ty} PFNV\exp(tya,z,zp)(1-DecayPFN)^{n(ty-tya)} \right] (1-\overline{FORICN(z,zp)})$$

$$where\ Ypf(ty,z,zp) = \{0,1\},\ PFNV\exp(ty,z,zp) \ge 0.$$

In addition to expanding the capacity of existing connections, eight new transmission lines are currently allowed in the model; ANG/NAM, ANG/DRC,

MWI/NMZ, MWI/ZAM, NMOZ/SMZ, SSA/NSA, ZAM/TAZ, and DRC/ZAM. Others can easily be added simply by entering the cost capacity and cost data in the appropriate tables. Values for *PFNinit(z,zp)* are found in Section 7 of Appendix III, values for *DecayPFN* are found in Section 13 of Appendix VII, and values for *FORICN(z,zp)* are found in Section 8 of Appendix VI. Further, it is required that total expansion respect the upper limit on expansion *PFNVmax(z,zp)* (found in Section 4 of Appendix III), and that expansion can take place only after initial construction:

$$\sum_{tya=1}^{ty} PFNVexp\left(tya,z,zp\right) \le PFNVmax\left(z,zp\right) \sum_{tya=1}^{ty} Ypf\left(tya,z,zp\right)$$

$$Ypf(\text{“}yr1\text{”},z,zp) = 0.$$

The GAMS format for: (a) the capacity constraint, equation *PFNmw*; (b) the requirement that expansion take place after initial construction, while at the same time insuring that expansion does not exceed the upper limit, equation *PFNmust*; and (c) the requirement that expansion/construction in one direction also means similar expansion construction in the other direction, equations *PFNdirect* and *Ypfdirect*, can be found in Appendix VII, Sections 11 and 12. Thus the only way users can enter new DC lines into the model is to set line loss in the counter flow direction at 0.999 in Table *PFNloss(z,zp)* (flows are from the row name to the column name). Use of 0 capacity in the counter flow direction alone will cause the objective function to reflect only ½ the cost of the line, rather than the full cost of the line.

Constraints that require that a new transmission line be started, or finished, before or during a given period can be found in Tables *Atlines(z,zp), Beflines(z,zp), Atlinesall(z,zp)*, and *Beflinesall(z,zp)*. Constraints that require that new projects not be started until after a given period can be found in Table *Aftlines(z,zp)*, all in Section 7 of Appendix III.

As in the discussion of generating capacity, discussion of the necessary modifications of these capacity equations to insure that a proper amount of transmission capacity be kept idle for possible flows of power from reserve generation held by country

z for country *zp*'s use will be postponed until the chapter describing the reliability equations.

3.10 Adjustments in Transmission Capacity to take into Account Generation Reserves held for and by others

In addition to the line by line capacity constraints just discussed, the model must require that there be sufficient transmission capacity between country *z* and *zp* to handle reserves held by *z* for *zp* – e.g., total transmission capacity between *z* and *zp* in year *ty*, the sum of old and new transmission capacity, must be greater than or equal to *Fmax(ty,z,zp)*; the sum of current old and new transmission capacity between *z* and *zp* as reported in 3.9 above ≥ *Fmax(ty,z,zp)*. This equation, called Equation *Fmaxtcapaj* is found in Section 13 of Appendix VII. Note that there is no need to require power flows plus *Fmax* to be less than transmission capacity, since, as is explained elsewhere, power flows include firm (*Fmax*) and non-firm power (*PF* minus *Fmax*, if > 0) see the discussion in Section 2 for more on this.

3.11 Pumped Storage Capacity Constraints

As in the case of hydroelectric dams, two types of capacity constraints are needed -- one for the maximum instantaneous rate at which water can flow out of the reservoir, and the other for the limit on the total amount of water which can be withdrawn over a period. The first is controlled by the size of the generating and pumping unit, the second by the volume of the reservoir.

Considering the maximum instantaneous amount of electricity which can be generated, if the parameters *PGPSOinit(z)* (Section 9, Appendix V), *PSOexpstep(z)* and *PSOmax(z)* (not now in the model) are the initial MW capacity, the size MW increment, and the maximum capacity respectively for existing generating/pumping equipment, and if *PSOexp(ty,z)* is the decision variable for capacity expansion - either positive or integer - and if *DecayPHO* (Appendix V, Section 9) is the percent decay parameter for pumped

units and *FORPHO(z)* the forced outage rate (Appendix V, Section 9), the MW generating capacity constraint for existing pumped storage can be written (unforced outage rates are assumed to be negligible):

$$PGPSO(ty,ts,td,th,z) \le PGPSOinit(z)(1-DecayPHO)^{n(ty)} + \sum_{\tau=1}^{ty} PSO\exp step(z)PSO\exp(\tau,z)(1-DecayPHO)^{n(ty-\tau)}$$

and the constraint that no more than the maximum amount can be added at a site:

$$\sum_{\tau=1}^{T} PSOexpstep(z)\,PSOexp(\tau,z) \le PSOmax(z)$$

The requirement that no more than the available volume of water in the reservoir can be used for power generation over any 24-hour period can be expressed as (equation *PHOcap* in Section 25 of Appendix VII):

$$\sum_{\tau=1}^{24} PGPSO(ty,ts,td,\tau,z) \le HDPSOmwh(z)$$

where *HDPSOmwh(z)* (found in Section 9 of Appendix V) is the MWh equivalent of the maximum volume of water available for generation in the existing reservoir over a 24-hour period.

As in the case of old thermal and hydro plants, the model allows old pumped hydro stations to be decommissioned at a user-specified time by entering the period in Table *FdecomPH(z)*, found in Section 9 of Appendix V.

The current version of the model assumes no additions are possible at the current pumped storage site in RSA - e.g., *PSOmax(z) = 0* and *PSOexpstep(z) = 0*. Thus pumped storage is limited to 1400 MW, the present capacity. Hence, in GAMS format, equations *PHOmw* and *PHOcap* are shown in Appendix VII, Section 25 with no expansion options. Thus, any additions to existing pumped hydro capacity must be treated as a new pumped hydro station, as described below.

The only difference between new and old pumped storage sites is the addition of a binary variable *Yph(ty,z,phn)* for each of the four possible sites preparation and

installation of additional capacity, and the substitution of *N* for *O* in the notation, the maximum MW capacity constraint is (equation *Phnmw*, Section 25 of Appendix VII):

$$PGPSN(ty,ts,td,th,z,phn) \le \Bigg[PHNinit(z,phn)\sum_{\tau=1}^{ty} Yph(\tau,z,phn)(1-DecayPhn)^{n(ty-\tau)} + \sum_{\tau=1}^{ty} PSNexpstep(z,phn)PSNexp(\tau,z,phn)(1-DecayPhn)^{n(ty-\tau)} \Bigg] \left[1-FORphn(z,phn)\right]$$

Table *PHNinit(z,phn)* can be found in Section 9 of Appendix V, as can values for *DecayPHN* and *FORphn(z,phn)*.

In addition, a constraint that prevents the cheaper additions from being added without the more expensive site preparation, and also limits expansion to its upper limit, once the initial site expense is complete:

$$\sum_{\tau=1}^{T} PSNexpstep(z,phn)\,PSNexp(\tau,z,phn) \le PSNmax(z,phn)\,Yph(ty,z,phn)$$

The maximum reservoir volume constraint, equation *PHNcap* (Section 25, Appendix VII) expressed in GWh, can be written:

$$\sum_{\tau=1}^{24} PGPSN(ty,ts,td,\tau,z,phn) \le HDPSNmwh(z,phn)\sum_{\tau=1}^{ty} Yph(\tau,z,phn)$$

(Table *HDPSNmwh* is found in Appendix V, Section 9.)

The current model allows construction of new pumped storage at four sites in RSA, but no expansion - e.g., *PSNmax(z,phn) = 0* for all *phn = 1, 2, 3, 4*, and hence *PSNexp(ty,z,phn) = 0* for all *ty*. Hence, in GAMS format, equations *PHNmw* and *PHN_ _one* are shown in Appendix VII, Section 25 with no expansion options.

Constraints that require new pumped storage projects to be started and completed during, before, or not before or at a given period can be found in Tables *AtPH(z,phn)*, *BefPH(z,phn)*, and *AftPH(z,phn)*, at the end of Section 9 of Appendix V.